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A method for the design of ship propulsion shaft systems

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A METHOD FOR THE DESIGN OF SHIP PROPULSION SHAFT SYSTEMS, by WILLIAM E. LEHR, JR. and EDWIN L. PARKER. Submitted to the Department of Naval Architecture and Marine Engineering on 20 May 1961 in partial fulfillment of the requirements for the Master of Science degree in Naval Architecture and Marine Engineering and the Professional Degree, Naval Engineer.

ABSTRACT

An investigation was conducted to establish a minimum span-length criteria for use in marine propulsion shafting design.

The investigation is conducted through computer studies of families of synthesized shafting systems. Each system is treated as a continuous beam carrying concentrated and distributed loads. In the studies span-length is systematically varied. The sensitivity of the study systems to alignment errors is investigated using reaction influence numbers. Relative insensitivity to misalignment is judged on the basis of limiting values of allowable bearing pressures and allowable difference in reactive loads at the reduction gear support bearings.

The results of this theoretical investigation indicate the desirability of increased values for span-length from those frequently found in present practice. Shaft systems with the following minimum span-lengths should be free from most problems resulting from normal alignment errors and the usual amount of bearing wear. (Span-lengths are expressed as length to diameter ratio)

For shafts with diameters 10 to 16 inches, L/D = 14
For shafts with diameters 16 to 30 inches, L/D = 12

In the conduct of the basic investigation several additional problems connected with shaft design were studied. A series of design nomograns for tailshaft sizing are derived from strength considerations. They are presented as a proposed aid for shaft design. The problem of fatigue failure of tailshafts, at the propeller keyway, is considered and a proposed method for corrective action is given.

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1.0 INTRODUCTION

1.1 Background

The years since 1940 have seen a complete evolution of the Navy's surface fleet, the advent of the super bulk carrier in the Merchant service, and the revolutionary change to nuclear propulsion in the submarine. These events, directly and indirectly, provided an impetus for a great deal of development and research in the field of marine turbines, reduction gears and propellers. Each prototype of these components has had the benefit of developmental research incorporated in the basic design procedure. In many cases research has been carried to the extent of building shore based test units to assist in the development. The result of this has been to make available to the shipbuilding industry efficient and trouble-free units. On the other hand, propulsion shafting, the connecting link in the propulsion system, has not been accorded the benefit of such research. Fortunately, most shaft systems designed using the existing criteria have provided excellent service. However, the need for further consideration of shafting design practices has made itself conspicuous in numerous ways, many of which have been covered com-

prehensively in recent presentations. Shafting effects on reduction gear alignment (1), the relatively high casualty rates of tailshafts (2), and the alignment problems of shaft bearings (3) are examples. In addition there are numerous operational reports of shaft seal failures and bearing failures.

1.2 Intent of Thesis Study

It is the authors' contention that many of the above problems will be alleviated, if the presently accepted design procedures are complemented by consideration of minimum bearing spacing and low frequency cyclic stresses. Thus, it is the intent of this thesis to present the effect of these two considerations on the design of a shaft system and provide a series of convenient nomograms and tabular data which will permit the designer to quickly develop the preliminary characteristics of a propulsion shaft system.

2.0 NOMENCLATURE

D - Outside diameter of shaft, (inches)

d - Inside diameter of shaft, (inches)

E.L. - Endurance limit of material, in air, (psi)

F.S. - Factor of safety

$F.S._d$ - Dynamic factor of safety. This factor accounts for the effects of dynamic loading and thrust eccentricity.

I_{x-y} - Influence number of bearing (x) on bearing (y), or the change in reaction at bearing (y) for a 1 mil deflection at bearing (x).

J - Polar moment of inertia of shaft section, (inches^4)

K_b - Stress concentration factor in bending. In shaft systems K_b is applied to shaft flanges, oil holes, etc. For well designed axial keyways $K_b = 1$.

K_t - Stress concentration factor in torsion. In shaft systems K_t is applied to flanges, oil holes and keyways.

K_1 - The percentage of steady mean torque which makes up the alternating torque. Maximum alternating torque occurs at the torsional critical speed. At speeds well removed from the torsional criticals, which should be the case for well designed systems, K_1 will range from 0.05 to 0.25 depending upon the hull configuration and proximity of the propeller to the hull, struts, etc. The selection of a value for K_1 must be based on the designers experience.

L_p - Moment arm of the propeller assembly. It is the distance from the center of gravity of the propeller to the point of support in the propeller bearing.

n - Ratio of inside to outside shaft diameters, d/D

Q - Mean or Steady torque

RPM	-	Revolutions per minute
R_x	-	Reaction in pounds at bearing (x)
ΔR_x	-	Change in reaction, in pounds, at bearing (x)
$R_{x_{sl}}$	-	Reaction in pounds at bearing (x), all bearings on straight line
S_b	-	Compressive stress due to bending, (psi)
S_c	-	Steady compressive stress, (psi)
SHP	-	Shaft horsepower
S_r	-	Resultant steady stress, (psi)
S_{ra}	-	Resultant alternating stress, (psi)
S_s	-	Steady shear stress, (psi)
S_{sa}	-	Alternating shear stress, (psi)
S_{sm}	-	Mean stress level of fluctuating steady stresses, (psi)
T	-	Propellor thrust, (lbs.)
W_p	-	Weight of propellor, (lbs.)
Y.P.	-	Yield Point of material, (psi)
Y_x	-	Deflection in mils of bearing (x) from straight line datum; + above datum, - below datum.

3.0 THE EFFECT OF SHORT SHAFT SPANS

3.1 General

At the present time classification rules in general make no mention of bearing spacing or of bearing loading, except to express the length of the bearing adjacent to the propeller as a function of shaft diameter. Most design procedures do limit indirectly the maximum bearing span by setting limits on allowable stresses, bearing load, and vibration considerations. However, as far as the authors have been able to determine, there are none that set a minimum on bearing span. Thus a system* such as shown in Figure 1 would satisfy the classification rules and by most design procedures would be considered a satisfactory shaft system.

That span is an important consideration in producing a satisfactory design is shown by a comparison of intended loads with the computed bearing loads in the system of Figure 1. The authors grant this considers only one particular case; however, the system is representative of certain current practices and vividly points

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* Equivalent Reduction Gear Diameter = 42.3"
Lineshaft Diameter = 21.9"
Tailshaft Diameter = 23.8"

up the problems encountered. To obtain the computed bearing loads, the shaft system composed of reduction gear, shaft, and propeller was treated as a continuous beam carrying distributed and concentrated loads. The influence line technique as developed by reference (4) and modified for use on the IBM-709 computer was applied permitting an analytical solution of the continuous beam problem. (See Appendix B)

3.2 Bearing Supports and Designed Reactions

It is assumed in the solution that the bearings act as zero clearance point supports at the mid-length of the bearings, with the exception of the after stern tube bearing. At the after stern tube bearing the support point is taken one shaft diameter forward of the aft end of the bearing. The effect of replacing the bearing surface by a point support does not significantly affect the results obtained, except in the case of the long stern tube bearings. For these bearings consideration must be given to the angle to which the stern tube has been bored and the state of weardown the bearing surfaces have attained. In the present case, it is assumed that the stern tube has been bored true to straight line datum and the bearing surfaces initially have sustained no weardown.

TYPICAL SHAFT SYSTEM

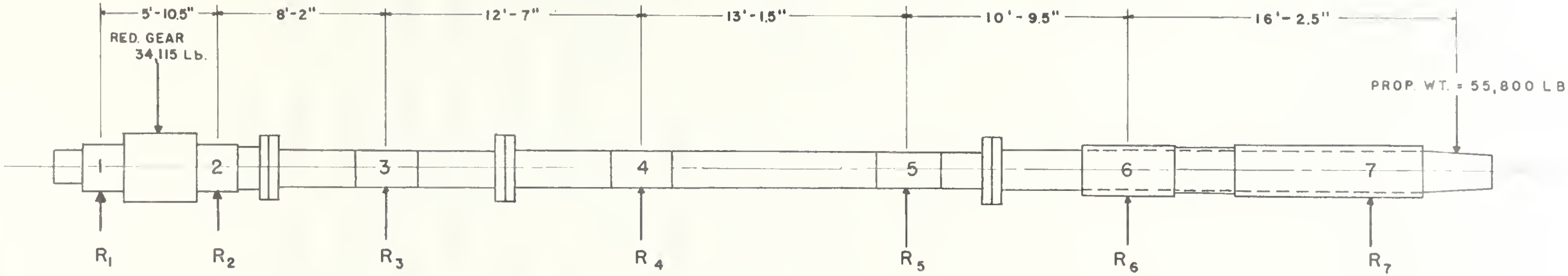


FIG. 1

SHAFT DEFLECTION CURVE

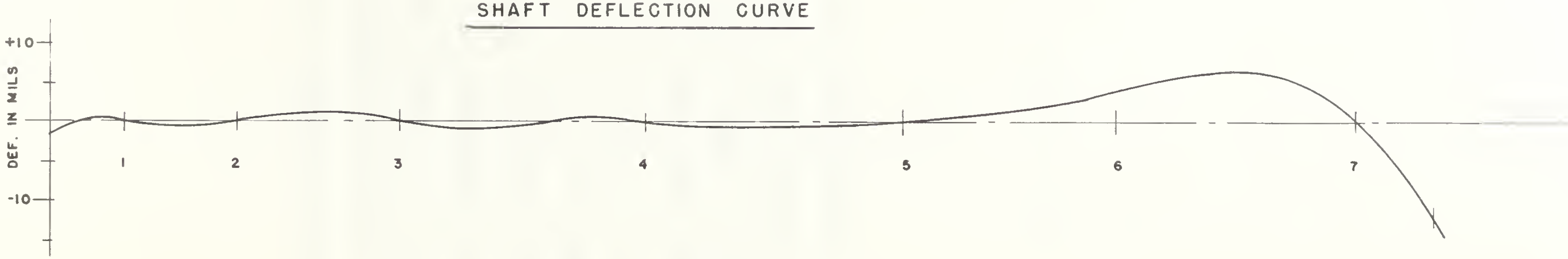


FIG. 2

Referring to the system shown in Figure 1, the intended bearing reactions with all bearings aligned on a straight line in the hot-operating condition were approximately as follows:

Bearing Number	1	2	3	4	5	6	7
Load (lbs)	35,000	35,000	17,000	17,000	17,000	25,000	75,00

3.3 Computed Bearing Loads

The initial calculation using the influence line technique showed the forward stern tube bearing No. 6 to be negatively loaded with all bearings on the straight line. In normal practice bearings have some diametrical clearance. Thus No. 6 journal would rise in its bearing. Effectively then, the bearing is not in the system, unless the rise is greater than the diametrical clearance. A second calculation was made with the support at the forward stern tube bearing omitted from the system. Table I consists of values of bearing reactions and influence numbers extracted from the computer output data for the second calculation.

TABLE I. - BEARING REACTIONS AND INFLUENCE NUMBERS

Influence Numbers (lbs. change per 0.001 inch bearing rise)

Brg. No.	1	2	3	4	5	6	7
1	2886.2	-5342.5	2761.1	-373.7	76.1	D	-7.2
2	-5342.5	10369.5	-5978.8	1166.9	-237.7	E	22.5
3	2761.1	-5978.8	4367.7	-1559.2	452.4	L	-42.9
4	-373.7	1166.9	-1559.2	1381.6	-734.2	E	119.0
5	76.1	-237.7	452.4	-734.2	575.4	T	-132.0
6	DELETED FROM SYSTEM						
7	-7.2	22.5	-42.9	119.0	-132.0	D	40.0
	Straight Line Bearing Reactions (lbs.)						
	33902.0	35545.4	15076.9	19156.7	17755.3	0.0	99834.3

3.4 Effect on After Bearings

Figure 2 shows a plot of the shaft deflections based upon data extracted from the computer output. It will be noted that at Bearing No. 6 the shaft is not resting on its support. The computer data indicated the journal is up 3.8 mils in the bearing. It is true that the journal will probably settle on its bearing due to compression of the after stern tube bearing under load, but at best it will be lightly loaded. Under these conditions of static

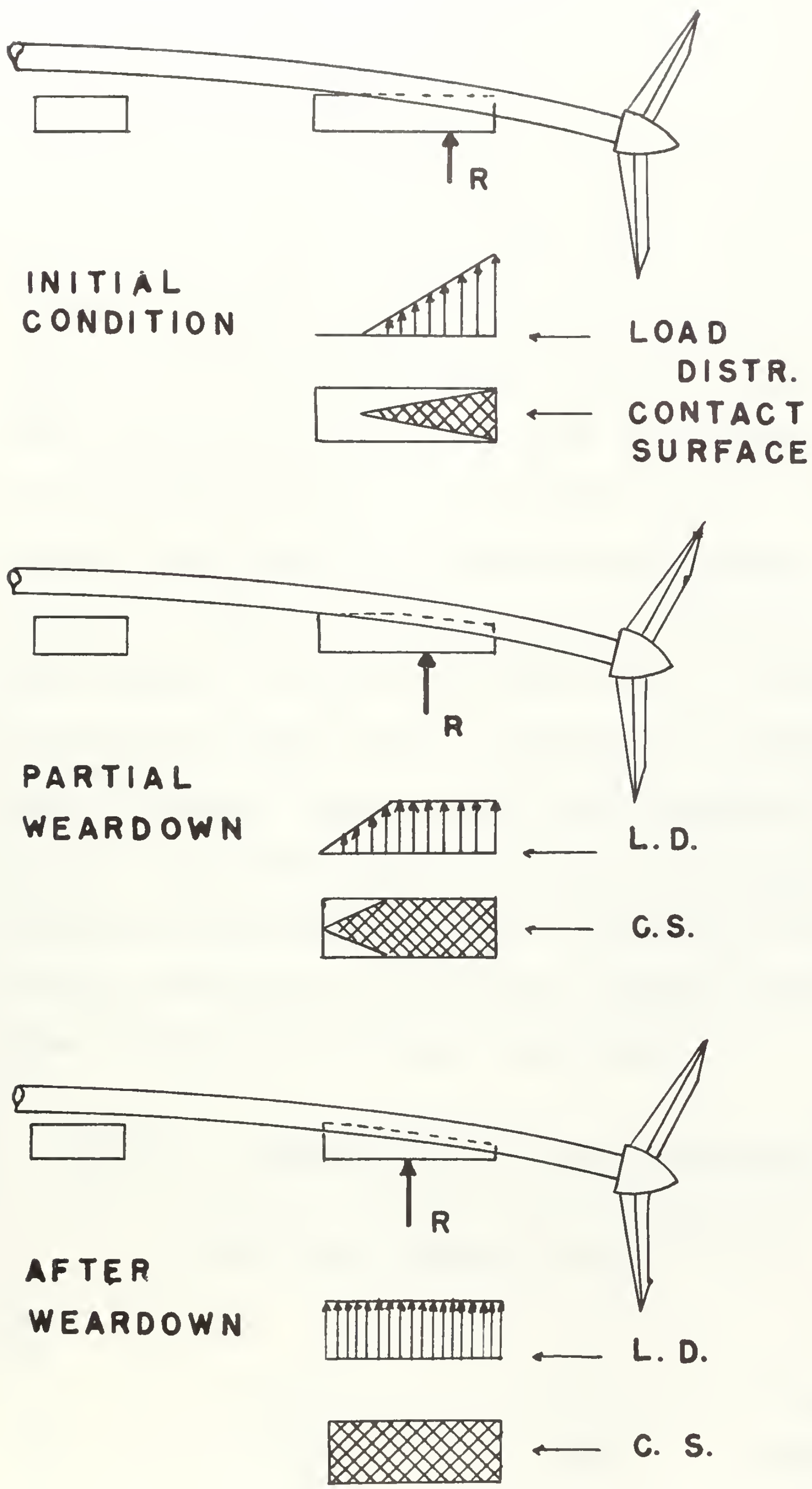

R
INITIAL CONDITION
LOAD DISTR.
CONTACT SURFACE
R
PARTIAL WEARDOWN
L. D.
C. S.
R
AFTER WEARDOWN
L. D.
C. S.

FIG. 3

loading any additional inertia loading caused by the ship working in a seaway could result in the journal pounding in the forward stern tube bearing. This condition in proximity to the shaft seal would make it impossible to maintain the integrity of the seal. In addition, as the after stern tube bearing begins to wear in, the equivalent point support shifts forward. This is shown in Figure 3. When the bearing has worn to the configuration of the elastic curve, there is uniform distribution of load on the bearing. The point support is then at the mid-length of the after stern tube bearing, as shown in Figure 3(c). A third calculation was made with all supports on a straight line and the support point of the after bearing at its mid-length. For this calculation No. 6 bearing was included in the system. Table II lists the values of bearing reactions and influence numbers for the after three bearing under these conditions.

TABLE II - BEARING REACTION AND INFLUENCE NUMBERS

Brg. No.	Influence Numbers (lbs. change per 0.001 inch bearing rise) 5	6	7
5	2140.9	-2115.6	781.4
6	-2115.6	2964.8	-1321.1
7	781.4	1321.1	642.4
Straight Line Bearing Reactions (lbs.)	31547.0	-46530.0	135124.0

A reasonable amount of weardown must take place before a uniform load distribution will occur on the after stern tube bearing. To determine the bearing loading under these conditions a value of weardown of 0.020 inches at the mid-length of the bearing was chosen. Using this value, Y_7 = -20.0 mils, and the influence numbers of Table II the calculated reactions at bearings No. 5 and No. 6 are:

(3-(1) $R_5 = R_{5s1} + I_{7-5}Y_7 = +31547 + (781.4)(-20.0) = 15{,}919.0$ lbs.

(3-(2) $R_6 = R_{6s1} + I_{7-6}Y_7 = -46530 + (-1321.1)(-20.0) = -20{,}108$ lbs.

The negative reaction at the forward stern tube bearing No. 6 indicates the journal will rise in that bearing and the reaction would be zero. Therefore setting $R_6' = 0.0$ the rise of the journal can be calculated.

(3-(3) $R_6' = R_6 + I_{6-6}Y_6 = 0.0 = -20{,}108.0 + (2964.8)Y_6$

$Y_6 = 6.8$ mils rise.

An adjustment is now made to the reactions at bearing No. 5 and No. 7 to reflect this rise of No. 6 journal.

(3-(4) $R_5' = R_5 + I_{6-5}Y_6 = 15{,}919.0 + (-2115.6)(6.8) = 1{,}533.0$ lbs.

(3-(5) $R_7' = R_{7s1} + I_{6-7}Y_6 + I_{7-7}Y_7 = 135{,}547.0 +$

$(-1321.1)(6.8) + (642.4)(-20.0) = 113{,}293.0$ lbs.

Not only has the forward stern tube bearing become unloaded but the adjacent lineshaft bearing No. 5 becomes lightly loaded. These conditions have negated any considerations the designer made, as far as strength, vibrations and bearing loads are concerned, based on all bearings carrying their designed load.

3.5 Corrective Action for Stern Bearings

It is possible to obtain the designed load reactions for these three bearings through use of a "faired curve alignment" (3). However, the magnitude of the influence numbers of Table II is unaffected by changes in vertical alignment. Since the magnitude is large in this area of the shaft, any appreciable error in initial setting or subsequent weardown of the stern tube bearing can cause unloading or overloading of the other bearings in the group. For example, the influence of bearing No. 5 upon itself is 2140.9 lbs./mil. Therefore a 5.0 mil error in setting would result in a 10,704 lbs. change in the reaction of the bearing.

Instead of using fair curve alignment, No. 6 bearing can be positively loaded by increasing the span length between bearings No. 6 and 7. Additionally, since the influence numbers are inverse functions of span length, a significant reduction in their values for bearings

No. 6 and 7 can be obtained. A softer tailshaft, relatively less sensitive to alignment errors and after stern tube bearing wear, is thus obtained.

Unfortunately, as the span between forward and after stern tube bearings is increased, the distance between bearings No. 5 and 6 decreases. This results in a corresponding increase in sensitivity to misalignment between No. 5 and No. 6. The most obvious solution to this dilemma is to remove the forward stern tube bearing entirely. In effect, this is physically what the system does. If the original design had considered such a possibility the stern seal would have been located near bearing No. 5. This of course assumes the stern tube could be lengthened to accommodate such action.

A comparison of Tables I and II shows a full order of magnitude decrease in the values of influence numbers can be obtained by such action. Thus the tailshaft is less sensitive to misalignment of bearing No. 5 and to weardown of the after stern tube bearing. Furthermore some positive loading has been achieved at the bearing next to the stern seal. As previously shown, it is small. An even greater span between the after stern tube bearing and the next forward bearing might be desirable from the standpoint of loading this bearing. Such an increase in span must be checked against strength, vibration and bear-

ing load requirements. Based on these it might not be possible to take such action without other adjustments. The point is that if the designer had considered a required minimum span length in the initial design stage, both strength and vibration criteria, as well as load and alignment criteria, could be met in this region.

3.6 Effect on Reduction Gear

Considering now the forward end of the system, additional alignment criteria must be met in the region of the reduction gear. In general, most gear manufacturers specify alignment and low speed gear bearing load conditions for satisfactory operation. These may take the form of a maximum difference in static fore-and-aft bull gear bearing reactions (1) or maximum and minimum acceptable bearing loads based on unit pressure. In any case the shaft system must be compatible with the requirements of the installed gear, or if direct drive, to the propulsion plant.

In the system under consideration it was the intent of the designer to have the gear bearing reactions approximately equal for straight line alignment. Looking at Table I, it is seen this intent is satisfied. However, for the gear to be on the straight line in the hot-

operating condition; it must be set some distance below the straight line in the cold-assembly condition. The distance which it is set below is made up of the rise of the bearings due to thermal expansion of the bearing supports and the rise of the journals in their bearings. This rise takes place when going from the cold-assembly condition to the hot-operating condition. Since assembly and operating conditions vary and the support structure is complex, some tolerance must be allowed in predicting this rise. Additionally, the erecting facility requires some tolerance in setting the gear. Therefore, the design must be such that the reduction gear and shaft combination is able to absorb some misalignment.

There are various criteria that could be used to evaluate the effects of misalignment. The authors chose to use the changes in reactions due to parallel displacement of the low speed gear bearings. Using the influence numbers of Table I, the reactions at bearings Nos. 1, 2 and 3 were calculated for various offsets using the following relationships:

(3-(6) $R_1 = R_{1s1} + I_{1-1}Y_1 + I_{2-1}Y_2 + I_{3-1}Y_3$

(3-(7) $R_2 = R_{2s1} + I_{1-2}Y_1 + I_{2-2}Y_2 + I_{3-2}Y_3$

(3-(8) $R_3 = R_{3s1} + I_{1-3}Y_1 + I_{2-3}Y_2 + I_{3-3}Y_3$

The influence of bearing No. 3 is included, so its effect may be calculated, when it unloads, and the journal begins to rise in the bearing.

The calculated values for the reactions are plotted in Figure 4. It will be noted that bearing No. 3 will unload when the offset is 4.5 mils above straight line datum. Similarly, at 7.0 mils below datum No. 2 bearing unloads. Now imposing the gear manufacturer's alignment criteria, which in this case was an allowable difference in fore-and-aft gear bearing reactions of 15,000 pounds, results in an allowable setting error of $\pm$ 2.0 mils, as shown in Figure 4. To attempt setting the gear with tolerances such as these would be completely unrealistic. If the gear could be properly positioned, it would be impossible to maintain these tolerances under service conditions.

Figure 5 is a plot illustrating the effect of misaligning No. 3 bearing in the system and very vividly shows the limited tolerances available for positioning of the spring bearings.

3.7 Corrective Action for Reduction Gear

To determine the effect of increased span between the after gear bearing and the following lineshaft bear-

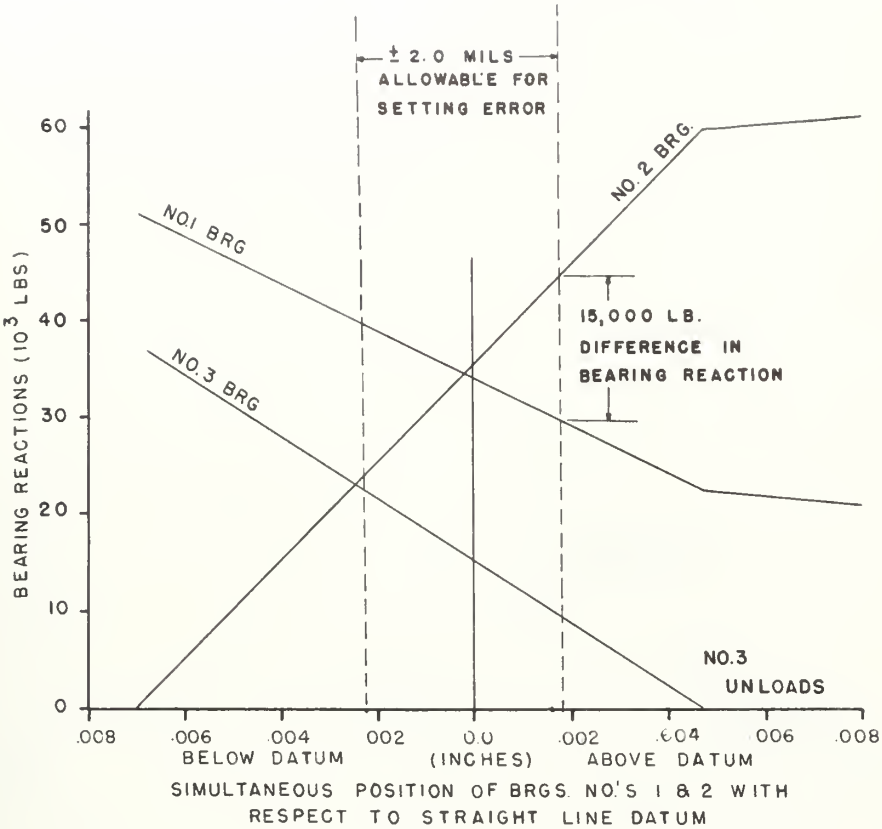

± 2.0 MILS
ALLOWABLE FOR
SETTING ERROR
NO. 2 BRG.
NO.1 BRG
NO.3 BRG
15,000 LB.
DIFFERENCE IN
BEARING REACTION
NO.3
UNLOADS
60
50
40
30
20
10
0
BEARING REACTIONS (10^3 LBS)
.008
.006
.004
.002
0.0
.002
.004
.006
.008
BELOW DATUM
(INCHES)
ABOVE DATUM
SIMULTANEOUS POSITION OF BRGS. NO.'S 1 & 2 WITH
RESPECT TO STRAIGHT LINE DATUM

FIG. 4

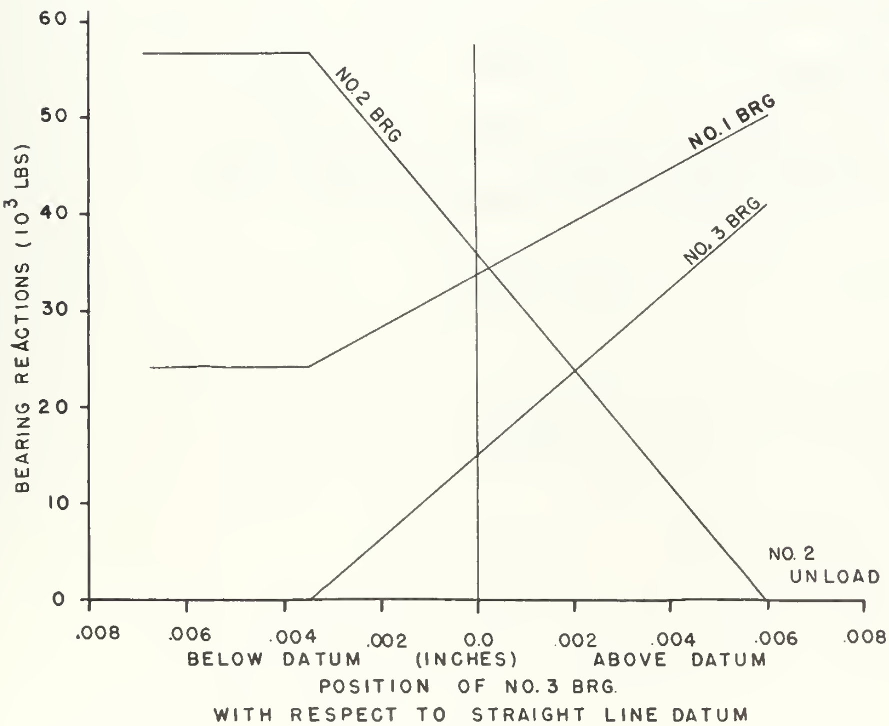
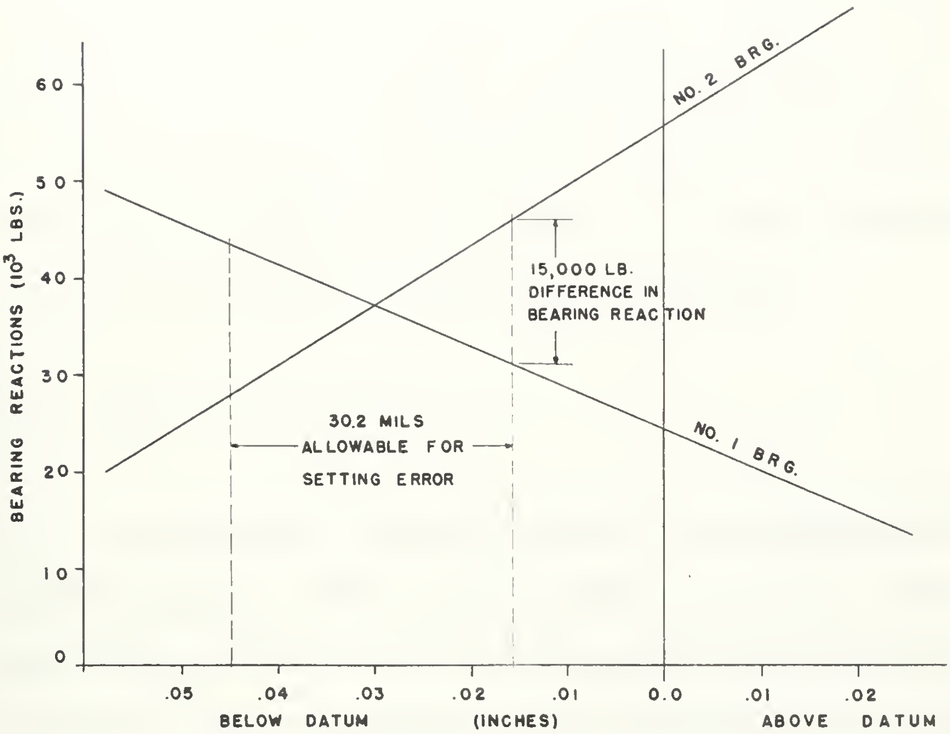

NO.2 BRG
NO.1 BRG
NO.3 BRG
NO. 2
UNLOAD
60
50
40
30
20
10
0
BEARING REACTIONS (10^3 LBS)
.008
.006
.004
.002
0.0
.002
.004
.006
.008
BELOW DATUM
(INCHES)
ABOVE DATUM
POSITION OF NO. 3 BRG.
WITH RESPECT TO STRAIGHT LINE DATUM

FIG. 5



SIMULTANEOUS POSITION OF BRGS. NOS. 1 & 2
WITH RESPECT TO STRAIGHT LINE DATUM,
NO. 3 BEARING NOT IN SYSTEM

FIG. 6

ing a fourth calculation was carried out with No. 3 bearing deleted from the system. Using the new influence numbers so obtained, the effect of parallel displacement of the gear bearings was determined. Figure 6 graphically depicts these results. It can be seen that a much more realistic tolerance of $\pm$ 15.0 mils has been obtained. As before, consideration must be given to strength, vibration and bearing loads before such a change could be made.

3.8 Results of Analysis

A particular system has now been analyzed and found unsatisfactory from several points of view. The system was actually built and the troubles predicted by the analysis were encountered. The significance of this illustration is that such a system can be built in complete accorance with both merchant and Navy design procedure. Normally, these procedures are tempered with the experience of the marine engineer, and many entirely satisfactory shaft systems have been designed and installed. It is now possible to develop an aid to the designer which removes the intuitive part of the design process.

4.0 CONSIDERATIONS FOR MINIMUM BEARING SPAN

4.1 Need for Minimum Span

During the analysis of a number of propulsion shaft systems, a definite relationship between length of bearing span and freedom from problems similar to those of the preceding example was noted. Troublesome systems had short bearing spans, while dependable systems had relatively long spans. All of the short span cases had high values of bearing reaction influence numbers, which resulted in large variations in bearing loads for only slight misalignments. This indicated the need for a minimum bearing span criteria to ensure a degree of insensitivity to alignment errors, and hence, better service operation of the system.

4.2 Alignment and Load Conditions

To establish a design criteria for minimum span length, it was necessary to determine the degree of misalignment to which the shaft system would be subjected, and to set load limits the system must meet under this misalignment. The authors considered the following as being most pertinent:

4.2.1) Bearing Loading. Upper limits on nominal pressure of 50 psi for oil lubricated shaft bearings, 35 psi for water lubricated wear down bearings, and 150 psi for pressure lubricated gear bearings were used. A lower limit on nominal pressure of 5 psi was used for all bearings.

4.2.2) Gear Alignment. It proved impossible for the authors to deduce any one criteria that would be universally accepted. Each gear unit, depending upon type, size and manufacture, has its own requirements. For purposes of establishing a requirement the authors used a maximum limit on the difference in static fore-and-aft bull gear bearing reactions. This data is listed in Table III.

4.2.3) Setting Tolerances. Here again, there is a wide diversity of opinion on necessary tolerances for the positioning of the system. Based on discussions with technical personnel and with some arbitrariness on the part of the authors, values which seemed to be both representative and realistic were selected. For the low speed gear bearings a value of $\pm$ 10.0 mils parallel displacement from the designed location was used. This allows a $\pm$ 5.0 mil error in predicting journal rise due

to thermal expansion of the bearing foundation and bearing reaction. In addition this allows the erecting facility a $\pm$ 5.0 mil error in positioning the gear. The same value of $\pm$ 10.0 mils vertical displacement from the designed location is allowed for the line-shaft bearings. No allowances were made for the wear down bearings. If the requirements for wear can be met, any mal-positioning of these bearings will also be satisfied.

4.2.4) Operational Wear. The only operational wear considered was that of the water lubricated bearings. Using the classification societies and U.S. Navy requirements as guides, a single value of 300 mils was selected for allowable weardown.

4.2.5) Foundation Flexure. In considering flexure of the foundation, i.e., the hull girder, it is assumed that in general it will follow a faired curve. If the system meets the foregoing requirements, it should adapt itself to the faired curve with no adverse effects (3). An exception to this can result from hard spots in the foundation structure. An example of the effect of hard spots is found in submarines. A bearing in the region of intermediate framing might deflect an appreciable

amount compared to a bearing in way of a deep frame when the hull is compressed due to submergence. For the present discussion, it is assumed that flexure of the hull imposes no additional requirements.

TABLE III

STUDY SYSTEM PARAMETERS

Shaft Diameter (inches)	10	12	14	16	18
Prop. Weight (lbs.)	11,250	15,000	18,500	25,000	35,250
Prop. Overhang (inches)	21	24	26	30	34
Gear Brg. Span (inches)	39.0	43.8	49.4	55.0	61.2
Equiv. Gear Dia. (inches)	27.0	28.0	32.2	34.0	36.7
Gear Shaft Dia. (inches)	11.8	14.0	16.3	18.6	21.0
Gear Weight (pounds)	15,400	23,200	33,000	43,500	55,500
Gear Face Length (inches)	25.0	28.3	31.9	35.5	39.5
Allowable Value of $\pm (R_1 - R_2)$ pounds	3,400	4,500	6,600	8,700	11,100
Propeller Mass Moment of Inertia Factor - $W_p r^2$	17.0×10^6	32.0×10^6	54.0×10^6	69.0×10^6	110.0×10^6

(continued)

TABLE III (continued)

STUDY SYSTEM PARAMETERS

Shaft Diameter (inches)	20	22	24	26	28	30
Prop. Weight (lbs.)	47,500	53,100	61,300	75,800	87,500	106,000
Prop. Overhand (inches)	38	43	47	51	55	60
Gear Brg. Span (inches)	67.3	78.5	80.8	90.8	101.5	112.7
Equiv. Gear Dia. (inches)	40.0	43.7	47.2	54.8	65.5	75.8
Gear Shaft Dia. (inches)	23.3	25.5	28.0	30.3	32.6	35.0
Gear Weight (pounds)	69,000	83,200	99,500	117,200	136,100	156,500
Gear Face Length (inches)	42.75	46.0	50.3	53.4	56.9	60.4
Allowable Value of $\pm (R_1 - R_2)$ pounds	13,800	16,640	19,900	23,440	27,220	31,300
Propeller Mass Moment of Inertia Factor - $W_p r^2$	163.0×10^6	147.0×10^6	181.0×10^6	235.0×10^6	302.0×10^6	382.0×10^6

5.0 EVALUATION PROCEDURE FOR MINIMUM SPAN

5.1 Development of Shaft Systems

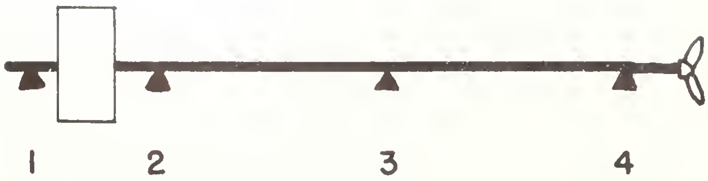
Using the computer program mentioned previously, a systematic study of families of shaft systems was carried out. The families of shaft systems were developed (See Appendix C) using the statistical data of reference (6) for the dimensions of the reduction gears and propellers. This data is listed in Table III. There are an infinite number of combinations of span length, shaft diameters, etc., that can make up a shaft system. In order to keep the number to be evaluated within reason and have the families related, the following limits were set:

1) Shaft diameters of 10.0 to 30.0 inches only were considered. The diameters were varied in two inch increments.

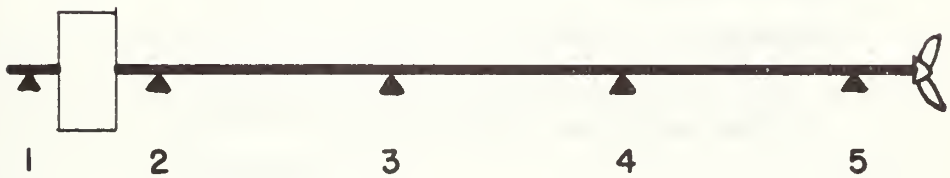
2) All span lengths between shaft bearings are equal.

3) Constant diameter shafts were used; i.e., the tailshaft and lineshaft have the same diameter.

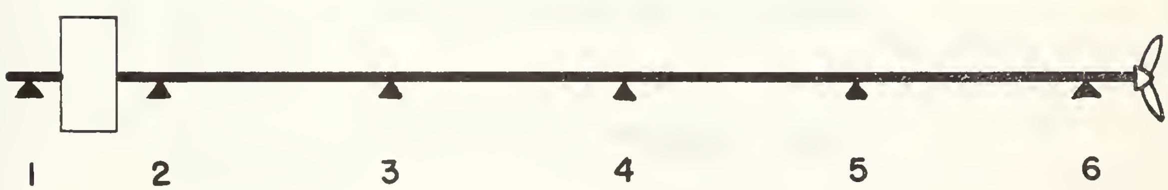
4) The effect of connecting flanges and shaft liners on system characteristics were ignored. The effect of these items is small compared to the major system components.



(A)



(B)



(C)

FIG. 7

5) Initially consider only two, three and four shaft bearing spans, see Figure 7.

6) Consider only span length to shaft diameter ratios of 10, 12, 14, 16, and 18. It would have been desirable to go to higher values and perhaps establish a definite upper limit on span length. However, the primary intent of the present study was the establishment of a minimum bearing span criteria. For this purpose the ratios selected were adequate.

5.2 Computer Output

Computer calculations were made for each case. The computer output consists of the following data:

1) Matrix of bearing reaction influence numbers.

2) Bearing reactions for straight line alignment.

3) Values of bending moments at selected points.

4) Values of shear stress at selected points.

5) Shaft deflections due to static loading.

Thus information for 165 related shaft systems was available for analysis by the authors.

5.3 Alignment for Comparison

For each case the table of influence numbers was applied in conjunction with the maximum values of allowed misalignment previously specified. In this manner the changes in bearing reaction were calculated for each system. These changes could have been applied to the straight line reactions and the results compared with the previously specified allowable values. For the straight line alignment of the system the gear bearing reactions are not normally equal. It was decided a more realistic base from which to evaluate the system was the alignment which causes equal gear bearing reactions. Equal reactions can be achieved by parallel movement of the gear bearings in the vertical direction. While this is not the only way in which to achieve equal reactions, it was the one which the authors considered more expeditious. The amount of offset was solved for as follows:

$$R_1 = R_2$$

$$Y_1 = Y_2$$

in the relationships

$$R_1 = R_{1s1} + I_{1-1}Y_1 + I_{2-1}Y_2$$

$$R_2 = R_{2s1} + I_{1-2}Y_1 + I_{2-2}Y_2$$

Combining and using the fact that $I_{2-1} = I_{1-2}$, by reciprocity, gives:

$$(9)\ Y_1 = Y_2 = \frac{(R_{2s1} - R_{1s1})}{(I_{1-1} - I_{2-2})}$$

Knowing the offset necessary to give equal gear bearing reactions, the reactions for all bearings were calculated.

5.4 Comparison Procedure

With the shaft system aligned in this manner, the procedure used for the comparison with allowable values was as follows:

<u>Condition I.</u> $\pm$ 10.0 mil parallel deflection of low speed gear bearings. The changes in reaction at the gear supports are

$$(10)\ \Delta R_1 = \pm 10.0(I_{1-1} + I_{2-1})$$

and

$$(11)\ \Delta R_2 = \pm 10.0(I_{1-2} + I_{2-2})$$

The difference in static fore-and-aft gear bearing reactions, remembering $I_{2-1} = I_{1-2}$, is then

$$(12)\quad (\Delta R_1 - \Delta R_2) = \pm 10.0(I_{1-1} - I_{2-2})$$

The change in reaction at bearing No. 3 is

$$(13)\quad \Delta R_3 = \pm 10.0(I_{1-3} + I_{2-3})$$

In a like manner the change in reactions at the other bearings in the system were calculated using the applicable influence numbers.

<u>Condition II</u>. $\pm$ 10.0 mil deflection of intermediate lineshaft bearings. The difference in static fore-and-aft gear bearing reactions is

$$(14)\quad (\Delta R_1 - \Delta R_2) = \pm 10.0(I_{3-1} - I_{3-2})$$

The change in reaction at bearing No. 3 is

$$(15)\quad \Delta R_3 = \pm 10.0(I_{3-3})$$

For systems with additional intermediate bearings the changes can be calculated in the same manner using the appropriate influence numbers.

<u>Condition III</u>. 300.0 mils of weardown of the stern tube bearing. The difference in static fore-and-aft gear bearing

reaction is

$$(16)\quad (\Delta R_1 - \Delta R_2) = -300.0(I_{4-1} - I_{4-2})$$

The change in reaction at bearing No. 3 is

$$(17)\quad \Delta R_3 = -300.0(I_{4-3})$$

The change in reaction at the after stern tube bearing is

$$(18)\quad \Delta R_4 = -300.0(I_{4-4}).$$

The above equations are for the two span system of Figure 7(a). For those systems with additional spans the appropriate influence numbers are used and the changes at the other bearings are calculated.

Before a system was considered as acceptable, it had to satisfy the following criteria derived from the previously outlined requirements:

1) $(\Delta R_1 - \Delta R_2)$ had to be less than the tabulated limit for difference in static fore-and-aft gear bearing reactions of Table III.

2) $(R + \Delta R)$, of the intermediate lineshaft bearings had to be such that the nominal bearing pressure is greater than 5.0 psi and less than 50.0 psi for a bearing with a length of

1.5 diameters. For example, in the case of bearing No. 3, this is

$$5.0(1.50D^2) < (R_3 + \Delta R_3) < 50.0(1.50D^2).$$

3) $(R + \Delta R)$, of the after stern tube bearing, had to be such that the nominal bearing pressure is greater than 5.0 psi and less than 35.0 psi for a bearing with a length of 4.0 diameters. For example, in the case of the two span system this is

$$5.0(4.0D^2) < (R_4 + \Delta R_4) < 35.0(4.0D^2).$$

These criteria had to be satisfied for Conditions I, II, and III individually. The authors did not evaluate the systems with all three conditions imposed simultaneously, and perhaps could be criticized for not covering the most stringent conditions. It is felt that the possibility of the maximum of all three conditions occurring at the same time is quite remote.

6.0 RESULTS OF MINIMUM SPAN EVALUATION

6.1 Three and Four Span Systems

The results of the evaluation showed that for three and four span systems, Figure 7 (b) and (c), the following satisfied all conditions:

1) Shaft diameter of 10.00 to 16.00 inches, a span length to diameter ratio equal to or greater than 14.

2) Shaft diameter of 16.00^+ to 30.00 inches, a span length to diameter ratio equal to or greater than 12.

6.2 Two Span Systems

For the two span system the limitation imposed on $(\Delta R_1 - \Delta R_2)$ was exceeded by the requirements of Condition III. This was true for all span length to diameter ratios investigated, with the exception of the shafts with diameters in the 26 to 30 inch range. For shafts with diameters of 26 to 30 inches, a span length to diameter ratio of 18 satisfied all conditions.

This does not mean that shorter spans cannot be used since several courses of action are available. First, the allowable weardown of the stern tube bearing could be reduced; second, accept a higher value of $(R_1 - R_2)$ than the value used by the authors; third, the initial alignment could be modified. The latter action would be the recommendation of the authors.

Basically, the procedure would be to initially align the system by offsetting the gear bearings to satisfy the following conditions:

1) The after gear bearings reaction R_2 greater than the forward gear bearing reaction R_1 by approximately 75% of the tabulated limit.

2) The unit pressure on No. 3 bearing equal to 15.0 psi; i.e., $R_3 = 15(1.5D^2)$.

Weardown of bearing No. 4 will increase the reaction at bearing No. 1 and decrease the reaction at bearing No. 2. As weardown proceeds the reaction at No. 1 and No. 2 will balance out; and after total weardown, the reaction at No. 1 will be greater than that at No. 2. For shaft systems 10.00 to 16.00 inches in diameter with span length to diameter ratios of 18 and systems 16.00^+ to 30.00 inches in diameter with span length to diameter ratios of 16, the final unbalance will be

approximately the tabulated limit and all other requirements will be satisfied.

6.3 Five Span Systems

The evaluation of several 5 span systems showed no further reduction of minimum span below that required in the 4 span systems was possible. The influence of adjacent bearings on one another remains nearly constant when the number of spans is increased above three. Based on this, it was deduced that the minimum span length for 4 spans is applicable for all systems having a larger number of spans.

6.4 Summary

These results are summarized in Table IV for systems with up to ten bearing spans.

6.5 Application of Results

The values in Table IV show only the minimum overall length for constant diameter shafting with equal span lengths. It is also applicable for use in systems with varying diameters and unequally spaced bearings. The diameter of the lineshaft will normally be smaller than

that of the tailshaft since the stress due to bending is lower in the lineshaft region. The shaft should be considered in two sections, and the applicable length to diameter ratio is obtained from Table IV by entering with the appropriate diameter. On the other hand, it may be necessary to use unequal bearing spans because of obstructions, bulkheads or framing. The only requirement is that the shortest span should have a length to diameter ratio equal to or greater than the value of Table IV, for the diameter used. Additionally the values of Table IV may be used in conjunction with hollow shafting by entering with an equivalent diameter, D', when

$$D' = (1-n^4)^{1/3} D$$

Table IV shows that before the transition from a two span system to one with three spans is possible, a minimum overall length of 36 and 42 shaft diameters is needed for the upper and lower diameter ranges respectively. This means span lengths of 18 and 21 shaft diameters would have to be used in the two span system. These may appear to be large when compared with present practices. However, studies by the authors for these particular systems indicated no difficulty from the standpoint of strength, vibration or bearing load, even with spans of 20 diameters for the upper range, and 22 diameters for the lower range

of shaft diameter. These are only guides for maximum allowable span lengths, and not hard fast rules. Each system is different and must be analyzed in light of the designers strength and vibration requirements.

It should be kept in mind that incorporated in the results are the specific limits of loading and gear geometry used by the authors. An attempt was made to use values which would approximate actual system parameters, thereby making the results directly applicable to the majority of systems encountered. There is always the exception when the foregoing results would have to be modified. For example, all other parameters being equal, a shorter distance between gear bearings would indicate a larger value for minimum span length. The authors feel that in most cases use of the values in Table IV will result in satisfactory operation in all respects.

TABLE IV

MINIMUM SHAFT AND SPAN LENGTH-DIAMETER RATIOS

D = 10.00 to 16.00 inches

Brg. Spans	2	3	4	5	6	7	8	9	10
L_{ms}/D	18	14	14	14	14	14	14	14	14
L_{mo}/D	36	42	56	70	84	98	112	126	140

D = 16.00^{+} to 30.00 inches

Brg. Spans	2	3	4	5	6	7	8	9	10
L_{ms}/D	16	12	12	12	12	12	12	12	12
L_{mo}/D	32	36	48	60	72	84	96	108	120

L_{ms} = Minimum Span Length

L_{mo} = Minimum Overall distance between the mid-length points of the after gear bearing and the aft stern tube bearing, for constant diameter shafts.

7.0 STERN TUBE BEARINGS

7.1 Advantage of One Stern Tube Bearing

The reader perhaps has questioned that weardown of only one bearing was considered. It is granted that in multiple screw arrangements with water-lubricated, intermediate, strut bearings, this would not be the case. The consideration of weardown at only one bearing does not invalidate the general requirements for minimum spans for such a system. On the other hand, it is proposed for all arrangements that the bearing adjacent to the stern seal be an internal oil-lubricated bearing. There are a number of very strong arguments in favor of making the change from the water-lubricated bearing used in present practice.

1) The use of a non-weardown bearing affords more positive control in positioning the shaft relative to the stern seal. Since no weardown takes place, the relationship with the stern seal is maintained during operation. This would alleviate many problems which the stern seal is noted for and would also increase the life of the seal.

2) Locating this bearing within the ship, as opposed to within the stern tube, permits

the use of a shorter stern tube. This is a consideration that arises in applying the minimum span criteria.

3) The use of an oil-lubricated bearing permits higher unit pressure and thus higher bearing loads without adverse effects.

4) For the lifetime of the ship maintenance costs for an oil lubricated bearing would be less, since bearing surfaces would not have to be replaced.

This arrangement in a single screw ship would result in only one stern tube bearing and was the reason weardown of only one bearing was considered. The evaluation studies showed this to be a feasible arrangement from the standpoint of bearing loads and certainly should result in better stern seal operation.

7.2 Location of Non-Weardown Bearing

This bearing should be located in the fore-and-aft direction as close to the stern seal as possible. To accomplish this it would be advantageous to have the tailshaft to lineshaft flange forward of the bearing. During

installation, it might be necessary to furnish temporary supports for the lineshaft. Additional thought must be given to removal of the tailshaft during routine dockings. This probably would be critical in the case of the two span system where clearing of the reduction gear might be difficult unless sufficient axial distance is allowed for pulling the tailshaft.

It is the opinion of the authors that systems incorporating the above arrangement will show a marked improvement in stern seal operation for the life of the ship.

8.0 LOW FREQUENCY CYCLIC STRESSES

8.1 Background

The problem of fatigue failure in way of the tailshaft keyway has aroused considerable interest in recent years, and a definitive study of the problem was reported in reference (7). In that report the consideration given the use of stress concentration factors in shafting design was of particular interest to the authors. It is their feeling that a great majority of tailshaft failures may be traced to the non-application of these factors in the initial design, notwithstanding the effects of corrosive atmosphere, fretting, etc. Furthermore it is felt that use of the concentration factors with so-called steady stresses is justified from the standpoint of low frequency fluctuations of the steady stresses.

The stress pattern in the tailshaft near the keyway is in general the result of steady shear, steady compressive, alternating shear, and alternating bending stresses. Steady shear (S_s) and steady compressive (S_c) are caused by mean shaft torque and thrust respectively. Alternating shear stress (S_{sa}) is the result of torque variations about the mean torque. Alternating bending stress (S_b) is caused by the lateral loading of the pro-

peller and shaft plus any applied moment. The procedure at the present time is to combine the steady stress components using maximum shear theory to obtain the resultant steady stress (S_r). Then in a similar fashion the resultant alternating stress (S_{ra}) is calculated. No consideration is given to the phase of the alternating components, since it is assumed they will be in phase periodically. The resultant stresses are then correlated to achieve a factor of utilization, or factor of safety, through use of the following relationship (8):

$$(19) \quad \frac{S_r}{Y.P.} + \frac{S_{ra}}{E.L.} = \frac{1}{F.S.}$$

or

$$(20) \quad \frac{\sqrt{(S_c^2 + (2S_s)^2}}{Y.P.} + \frac{\sqrt{(k_b S_b)^2 + (2k_t S_{sa})^2}}{E.L.} = \frac{1}{F.S.}$$

8.2 Stress Concentration

In a shaft under load, the stress level at sharp corners, pits, oil holes, etc., is known to be greater than that of the applied unit stress. The theoretical stress concentration factors are predicted values of stress magnification around such discontinuities in the shaft structure. The concentration factors are derived from the geometry of the stress raisers and the type of loading

applied. It has been shown in laboratory tests that the presence of a stress raiser has only a minimal effect on the level of continuously applied steady stress that will cause failure. On the other hand, when an alternating stress is applied, the level of applied stress for failure is significantly reduced in specimens with stress raisers.

The theoretical concept for this phenomenon relates ability to carry an applied load to the distribution of stress around the stress raiser. For steady stress the magnification of applied stress is believed to cause some localized yielding in the region of the stress raiser. Because of the yielding a redistribution of the stress pattern occurs, effectively reducing the maximum level of stress in the pattern (9). The applied load can then be carried even though some yielding may occur. In the case of alternating stresses only highly localized yielding takes place. An associated high stress level results. Under repeated load application fatigue failure will occur. Equation 19 takes these effects into account by incorporating the stress concentration factors with the alternating stresses. Its application has proven satisfactory in the design of many power transmission systems which operate at a continuous level of steady stress with

a superimposed alternating stress. At the present time this is the procedure used as strength criteria by the U. S. Navy to determine shaft diameter (10). Furthermore, many merchant designs incorporate a similar criteria as a check on the adequancy of the required diameters specified by the classification societies.

The application of the foregoing procedure to ship shafting, however, does not always provide a fail-safe system. In particular focus attention on the propeller keyway. If it is assumed that the keyway is well designed and follows an easy taper in the axial direction, a stress concentration factor in torsion only would be applied to the alternating shear stress component. Theoretically, and within the accuracy of predicted values for the steady mean stress and alternating stress, a tailshaft with a definite factor of safety should be the result of applying equation 20. Yet many tailshafts designed for strength in this manner suffer fatigue crack failures in the region of the keyway.

8.3 Steady Stress Reversal Theory

It is suggested that these failures are the result of reversals of the large steady stresses. This is a factor which is not accounted for in the present procedure. Of course fluctuations of the steady stresses occurs in any

power transmission system each time it is started and stopped. However, there are few systems operating at a comparable load level, that are started ahead, stopped, backed down and started ahead again as often as ships' shafting. It is theorized that each time a ship is started, stopped, reversed, etc., constitutes reversal of the supposedly steady stresses. Thus the steady stresses actually must be considered as alternating stresses of low cyclic frequency. It is hypothesized that stress concentration factors must be applied to the steady stresses to eliminate fatigue failures because of this.

The graphical representation of the above remarks might illustrate the hypothesis more clearly. Figure 8(a) is an assumed fatigue curve for a metal similar to class 2 steel. The endurance limit shown corresponds to the level of mean stress applied. Superimposed on this curve is a plot of the stress levels considered in the direct application of Equation 20. It is seen the maximum applied stress is equal to the sum of the continuously applied steady stress plus the alternating stress. It must be remembered that the fatigue curve is not an analytical curve, but is the result of experimental data. The level of the endurance limit is a function of the magnitude of the applied mean stress. Figure 8(b) illustrates the same

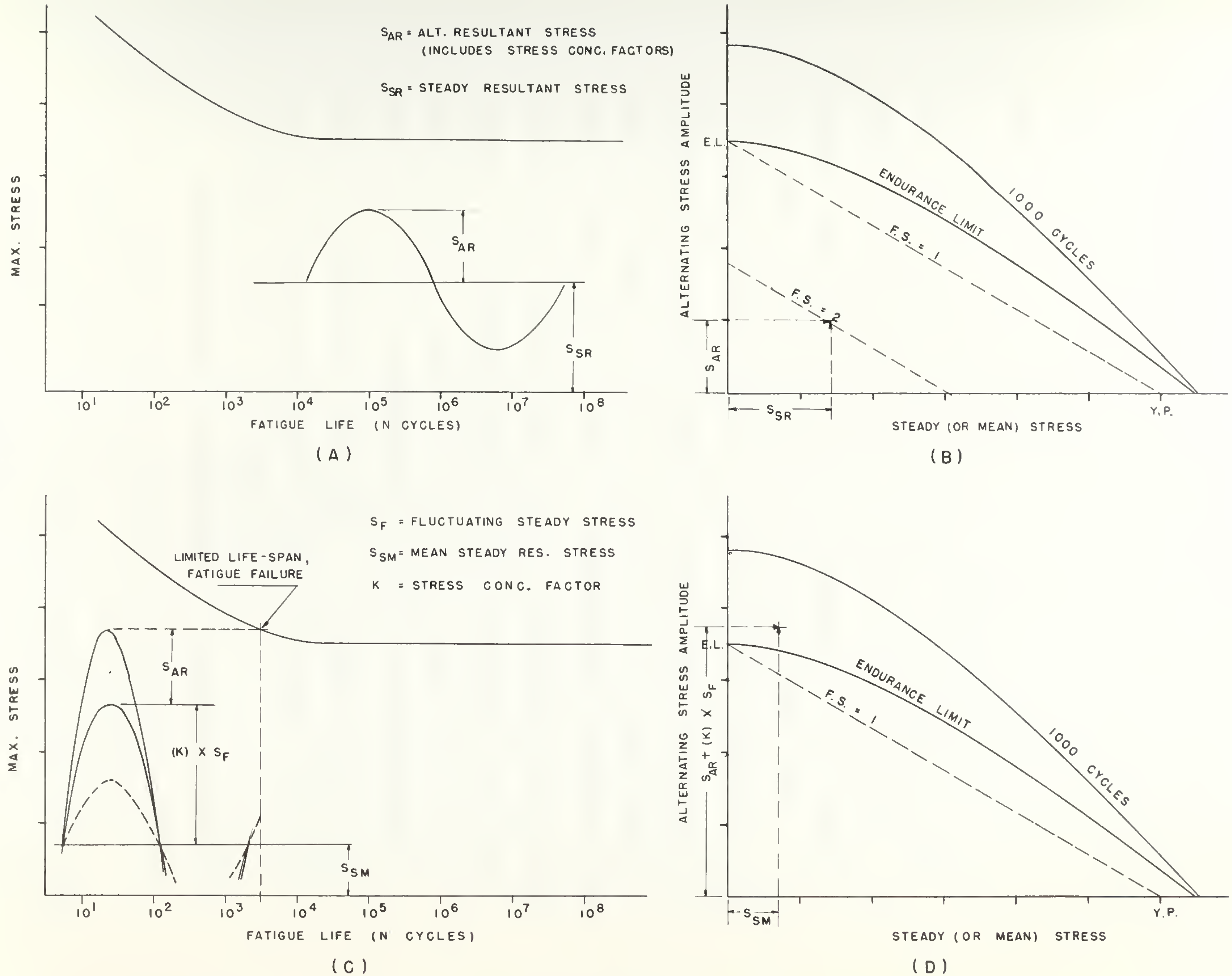


FIG. 8 - EFFECT OF LOW CYCLE STEADY STRESS REVERSAL

stress conditions as represented on a modified Goodman diagram and is the graphical representation of Equation 19.

The effect of low frequency variations of the steady stresses is similarly shown in Figures 8(c) and (d). A ship operates more often in the ahead condition than in the astern so there will be some mean stress level about which the steady stress fluctuates. The value of mean stress has been arbitrarily selected in the figures. If stress concentration factors are applied to the portion of steady stress in excess of the mean value, the fluctuating curve shown in Figure 8(c) is obtained. The original alternating stress is still present and must be included causing a further increase in the magnitude of the maximum applied stress. The cumulative effect of the fluctuating steady stress and the alternating stress may result in a maximum applied stress in excess of the endurance limit. Rather than having a system with an indefinite life, the designer may find his system has a definite predictable life span. A fatigue failure will occur as soon as sufficient starts and stops, or cycles of steady stress, have taken place.

Depending upon the magnitude of the maximum applied stress, fatigue failure conceivably could occur after a few hundred or more cycles. It is not difficult to visualize a ship's shafting system accumulating this many steady stress reversals in a few years operation. Hence,

an unpredicted early failure due to fatigue may occur.

8.4 Application in Design

The most difficult problem in the attempt to incorporate the foregoing in the present design procedure is the determination of a mean steady stress level about which the steady stress will vary. A statistical study of a number of ship's bell books coupled with a shaft stress analysis could provide the answer. Once the mean stress level has been determined, the appropriate stress concentration factors can be applied, and the shaft may be designed so as to avoid fatigue failure during the life of the ship.

Summarizing, it is hypothesized that the factor of safety, a better term would be factor of utilization, predicted by the presently used working stress equation may be in error, since it is not based on a consideration of the low frequency variations of steady stress. This should not be construed as questioning the validity of the equation but rather the validity of the term steady stress.

It is theorized that this error can be corrected by applying stress concentration factors to the fluctuating steady stresses. This results in the working stress equation taking the form

$$(21)\quad \frac{1}{F.S.} = \frac{\sqrt{(1-p)(S_c^2+4S_s^2)}}{Y.P.} + \frac{\sqrt{\left[(k_b)(S_b+pS_c)\right]^2 + \left[(2k_t)(S_{sa}+pS_s)\right]^2}}{E.L.}$$

$$p = \frac{(\text{Maximum Steady Stress}) - (\text{Mean Value of Steady Stress})}{(\text{Maximum Steady Stress})}$$

This equation will require some increase in diameter over that specified by the presently used equation 20 to achieve the same factor of safety. Through the use of this modified form of the equation a designer should be able to provide, barring other effects, shafting with a more accurately predicted service life.

9.0 SIZING OF THE TAILSHAFT

9.1 Present Procedure

The maximum stress level in a shaft system can be expected in the region of the tailshaft. This is a result of the large cantilevered propeller weight introducing bending stresses. Thus in propulsion shaft design the first step normally involves the selection of an adequate tailshaft diameter. At the present time the designer must estimate an initial diameter or compute a minimum required diameter by use of a classification rule. A strength calculation based on that estimate is made to ensure that the cross section can carry the expected load with some factor of safety. If the initial estimate was good and the desired factor of safety is obtained no further calculation is required. This is not always the case and several trials may be required.

It is possible to carry out an analytical solution for shaft diameter using Equation 20 and the relationships for the steady and alternating stresses as a function of diameter and the applied load. Unfortunately, such a direct calculation requires the solution of a sixth order polynomial in diameter (D) which is little improvement over the trial and error procedure. A simple solution of

the polynomial is available through use of nomographic techniques. In this manner a direct solution is available.

9.2 Development of Design Nomograms

In the development of the nomograms it was assumed that the designer would have the following two sets of parameters available. The first are characteristics specified by the particular hull and ship's speed.

1. Shaft Horsepower
2. Shaft RPM
3. Propeller Weight
4. Propeller Thrust
5. Propeller Overhang

The second set of parameters are design variables and are dependent upon material used and the designer's criteria and experience.

1. Endurance limit of material
2. Yield point of material
3. Ratio of inside to outside diameter
4. Stress concentration factor for bending
5. Stress concentration factor for torsion
6. Percentage of steady mean torque which is equivalent to alternating torque

7. Desired factor of safety
8. Dynamic factor of safety to account for inertia loading and eccentricity of thrust.

With these known parameters, the stresses in the tailshaft can be expressed as functions of the required diameter.

Steady Shear stress,

$$(22) \quad S_s = \frac{321,000(SHP)}{(1-n^4)\ D^3(RPM)}$$

Steady Compressive stress,

$$(23) \quad S_c = \frac{1.273(T)}{(1-n^2)D^2}$$

Alternating Shear stress,

$$(24) \quad S_{sa} = K_1(S_s)$$

Alternating Compressive stress due to bending,

$$(25) \quad S_b = \frac{10.187(F.S._d)W_pL_p}{(1-n^4)\ D^3}$$

Combining the above with Equation 20 and rearranging results in a sixth order polynomial in diameter as a function of the known parameters.

$$(26)\quad D^6 - N(D^3) - L(D^2) + M = 0.0$$

where;

$$N = \frac{2(F.S.)}{(1-n^4)(E.L.)}\sqrt{(10.187K_b(F.S._d)W_pL_p)^2 + (642{,}000\ K_tK_1SHP/RPM)^2}$$

$$L = \left[\frac{1.273\ T(F.S.)}{(1-n^2)(Y.P.)}\right]^2$$

$$M = \frac{N^2}{4} - \left[\frac{642{,}000(SHP)(F.S.)}{(1-n^4)(RPM)(Y.P.)}\right]^2$$

Figure A-6 is the graphical solution of Equation 26. To enter the figure values of L, M, and N must be known. They can be calculated directly, however as a time saver, the peripheral diagrams Figures A-1 to A-5 have been developed to facilitate a simple graphical solution.

In the development of the diagrams no limitations as far as possible combinations of parameters were specified with the exception of Figure A-6. For Figure A-6 an upper limit was placed on the values of M and N. This was done since it is inconceivable that a material with the lowest value of yield point would be used with a combination of the highest SHP, propeller weight, etc.

It should be noted that the diagrams as developed do not include a consideration of the effects of steady

stress fluctuations. It would have been most desirable to have included this effect. The authors decided in the absence of an accurate estimate of the mean steady stress level not to include it in the preliminary design stage. It was the opinion of the authors that inclusion of this effect should be in the form of a check on the adequacy of the design. In which case, the stress components can be calculated using the diameter selected. Then using Equation 21 and the designer's estimate of the mean steady stress level a factor of safety can be calculated. A comparison of this factor of safety with the intended factor used in the selection of the diameter would indicate the adequacy of the design. In this manner consideration is given to both high and low frequency cyclic stresses.

10.0 CONCLUSIONS AND RECOMMENDATIONS

The power transmission system for a ship is an important, integral part of the propulsion plant. As such it requires a comprehensive design procedure to ensure an optimum trouble-free system. To obtain a good design the shafting, bearings, reduction gear, and propeller must be considered as a single integrated unit. Specifically, a recommended procedure would include:

1. The integrated system should be treated as a continuous beam carrying both distributed and concentrated loads and carried by point supports at the bearing locations. Using this arrangement a solution of the continuous beam problem should be obtained with particular attention to support reactions, deflections and bending moments.

2. Minimum span lengths, or maximum number of support bearings, should be selected on the basis of insensitivity to initial misalignment errors and wear-down of the water-lubricated bearings. Any method used to judge the degree of insensitivity should consider the effects of misalignment on allowable bearing pressures and change in reactions at the reduction gear bearings.

Table IV can be used as a guide in span length selection. It is recommended that for a given shaft diameter and overall length, the values listed in Table IV be considered as minimum span lengths.

3. In general each design should include a comprehensive study of strength and vibration characteristics. Both required shaft diameter and maximum span length are set by strength and vibration requirements. Maximum span lengths of 20 to 22 diameters are possible. However, each design must be checked to ensure that critical whirling frequency criteria are met if spans of this length are used. Required tailshaft diameter can be obtained through an application of equation (20) or through use of the procedure of Appendix A.

4. In connection with required shaft diameter, consideration should be given to the adverse effect of low frequency fluctuations of the supposedly steady stresses. To aid such a consideration, it is recommended that a statistical study be undertaken to facilitate prediction of the mean level of steady stress. The study could consist of a survey of the bell books of several types of ships now in operation. A quantitative summary of ahead and astern engine orders would then be available from which predicted levels of mean stress could be derived. With this

information it would be possible to correlate endurance limit, low cycle stress fluctuations and maximum applied stress.

5. Consideration should be given to the installation of a non-weardown bearing adjacent to the stern seal. This bearing should replace the presently used water-lubricated bearing. In this manner more definite support in the seal area will be provided.

6. The theoretical investigation for minimum span lengths revealed that extra difficulties may be expected with two span systems. These close coupled systems should be avoided whenever possible. It would be desirable to use overall system lengths suitable for three spans in the interest of design simplicity. Whenever two span systems must be used, it will be necessary to specify an optimum system alignment as well as minimum span length to get the most desirable shafting system.

If the recommended design procedure is carried out it should be possible to design shaft systems which will provide optimum operating characteristics. They will be immune to excessive gear tooth wear, stern seal difficulties, and support bearing problems. In addition some improvement should be obtained as far as tailshaft liner and fatigue failures are concerned.

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(11) E. Panagopulos, "Design-Stage Calculations of Torsional, Axial, and Lateral Vibrations of Marine Shafting", Transactions SNAME, 1950.

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A P P E N D I X

APPENDIX A

RECOMMENDED PRELIMINARY DESIGN PROCEDURE

Set up the following tabular form:

Hull Parameters

1. Shaft horsepower .... ______hp.
2. RPM .... ______RPM
3. Propeller Weight (W_p)..................... ______lbs.
4. Thrust (T) .... ______lbs.
5. Propeller Overhang (L_p) .... ______ins.
6. Overall length (L_o) .... ______ins.

Design Parameters

1. Diameter ratio (n) .... ______
2. Percent Steady Torque (K_1) .... ______
3. Stress Concentration bending (K_b) ...... ______
4. Stress Concentration torsion (K_t) ...... ______
5. Yield Point of Material (Y.P.) .... ______psi
6. Endurance Limit of Material in Air(E.L.) ______psi
7. Dynamic Factor of Safety desired .... ______
8. Factor of Safety desired .... ______

Enter Figure A-1 with (Y.P.) and (n) connect and mark the V-Scale, then connect (T) and (F.S.) and mark U-Scale. Connect V and U and read L-Scale. L = ______

Enter Figure A-2 with (K_t) and SHP connect and mark intersection on U-Scale. Connect U with (K_1) entry and mark intersection on SHP scale. Connect this point with the RPM entry and read A-Scale. A = __________

Enter Figure A-3 with (K_b) and F.S.$_d$) connect and mark the U-Scale. Connect this point with (L_p) and mark intersection on V-Scale. Connect this point with (W_p) and read B-Scale. B = __________

Enter Figure A-4 with A and B, from intersection of A and B follow circle to left-hand vertical scale. Connect this point with (F.S.) and mark intersection on S-Scale. Connect S with (E.L.) and read C-Scale. C = __________

Enter Figure A-5 with (Y.P.) and RPM connect and mark S-Scale. Connect (F.S.) and SHP and mark T-Scale. Connect T and S Scale and read D Scale. D* __________

---

* This is a dummy variable and should not be confused with Diameter (D).

Perform the following calculations:

1. $n^4 =$ ________.

2. $(1-n^4) =$ ________.

3. $N = C/(1-n^4)$ ________. $N =$ ________

4. $N^2/4 =$ ________.

5. $\dfrac{D}{10(1-n^4)}$ ________.

6. $E^2 =$ ________.

7. $M = N^2/4 - E^2$ ________. $M =$ ________

Enter Figure A-6 with L and N and connect with a straight line. Locate the intersection of this line with the value of M. Follow the vertical line from this point and read the value for Tailshaft diameter. Diameter = ________

Divide L_o by the Diameter, L/D = ________

Enter Table IV of the paper in the appropriate diameter range and determine the number of bearing spans that can be used. This is only a first approximation, since the designer will want to adjust the line-

shaft diameter to meet his own criteria of strength and vibration. However, the requirements for minimum span must still be met for the diameter used.

The above procedure did not consider the low frequency cyclic stresses. If the designer wishes to incorporate this consideration he may do so by use of Equation 21.

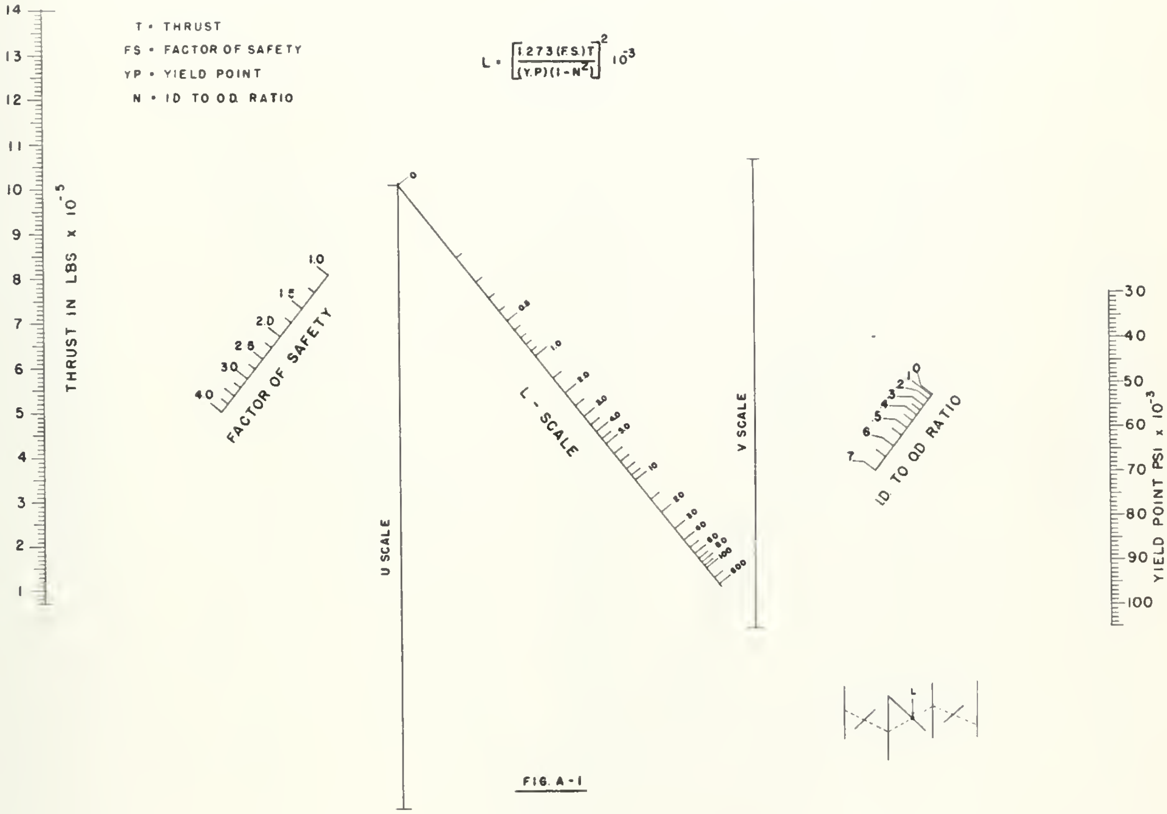

T = THRUST
FS = FACTOR OF SAFETY
YP = YIELD POINT
N = ID TO OD RATIO
$L = \left[\frac{1.273 (F.S.) T}{(Y.P.)(1-N^2)}\right]^2 10^{-3}$
THRUST IN LBS x 10^{-3}
FACTOR OF SAFETY
U SCALE
L - SCALE
V SCALE
ID TO OD RATIO
YIELD POINT PSI x 10^{-3}

FIG. A-1

$$A = \frac{642{,}000\ SHP(K_T K_I) \times 10^{-6}}{RPM}$$

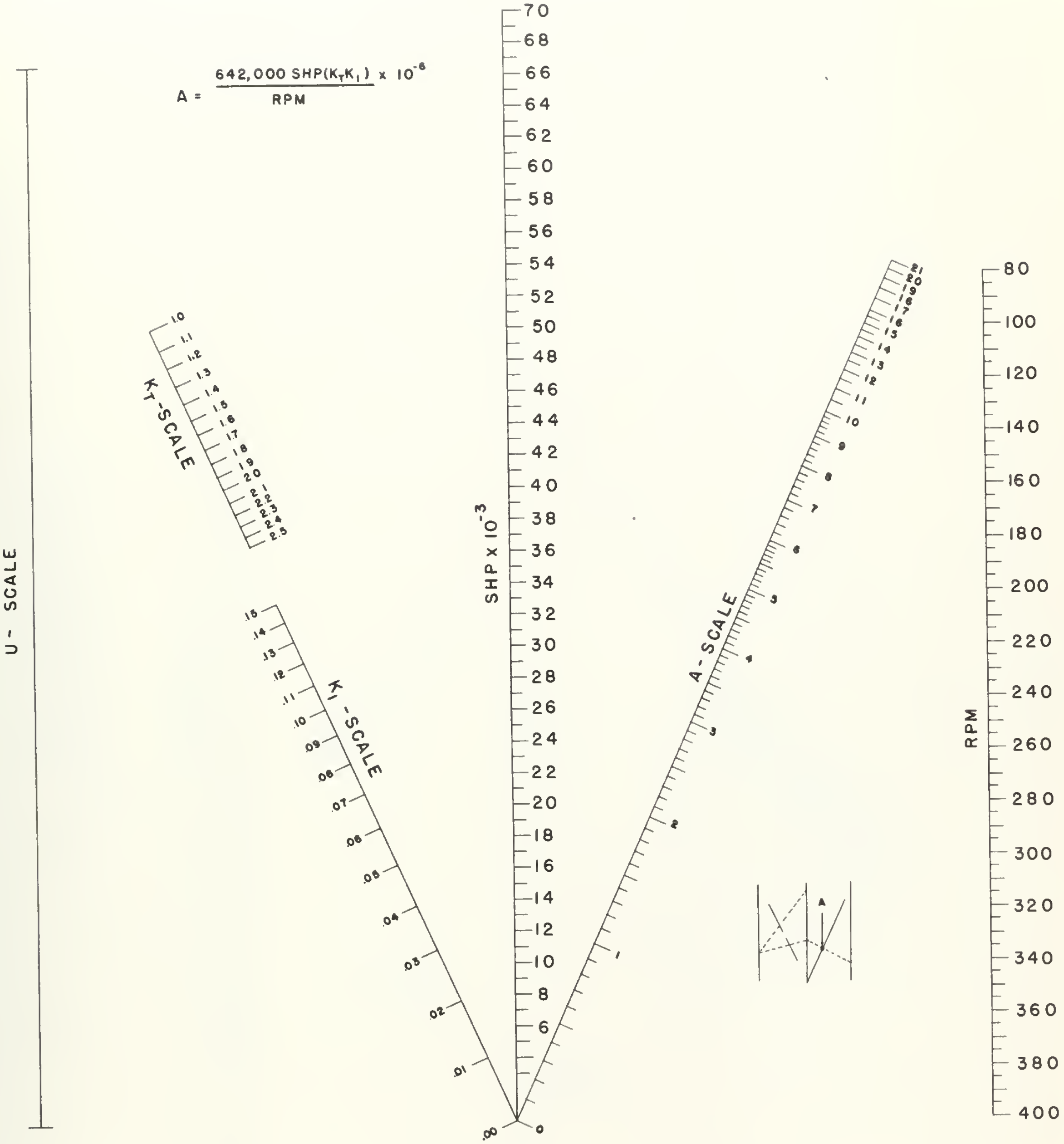
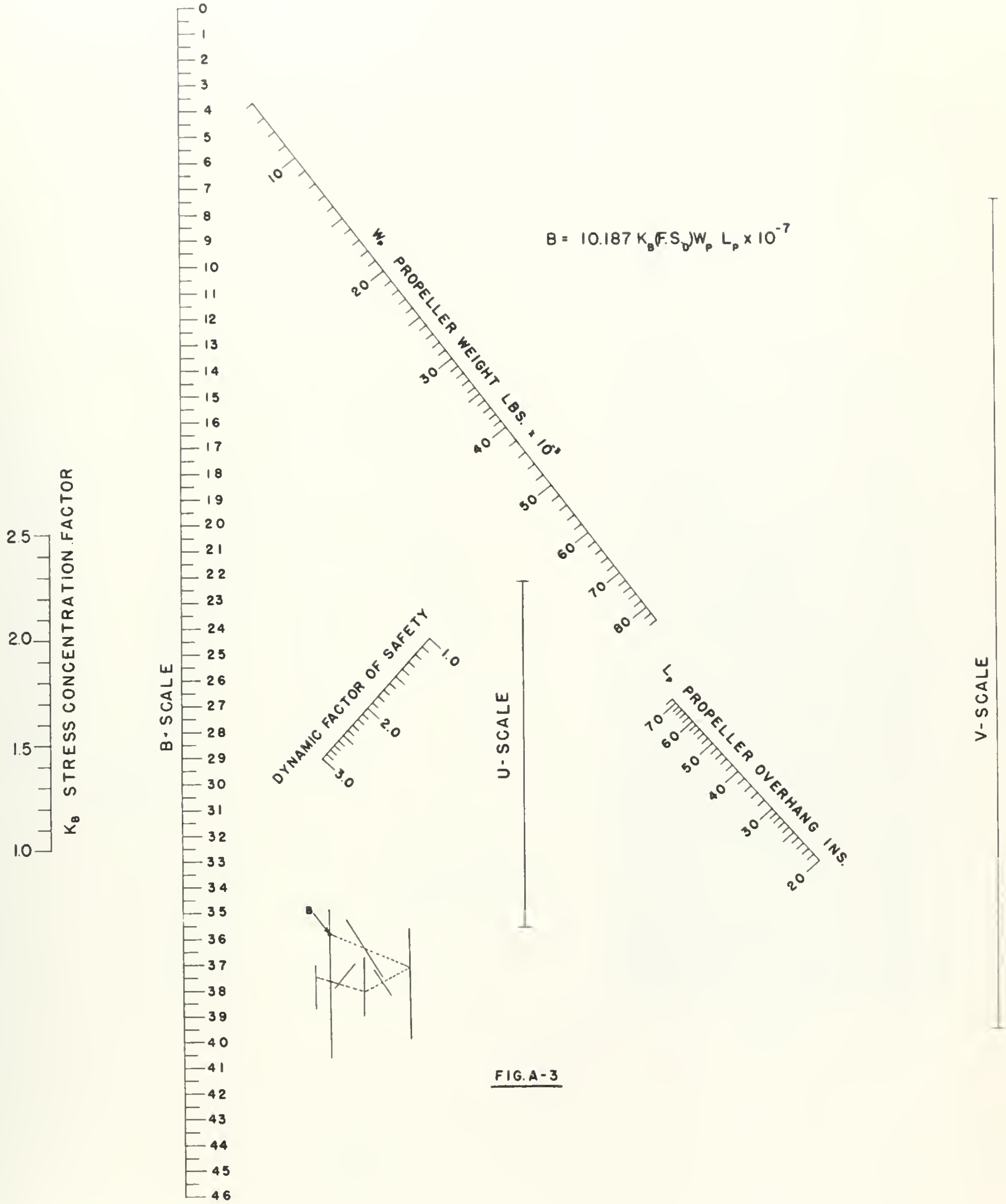


FIG. A-2



B = 10.187 K_B (F.S._D) W_P L_P x 10^-7
K_B STRESS CONCENTRATION FACTOR
B-SCALE
W_P PROPELLER WEIGHT LBS. x 10^3
DYNAMIC FACTOR OF SAFETY
U-SCALE
L_P PROPELLER OVERHANG INS.
V-SCALE

FIG. A-3

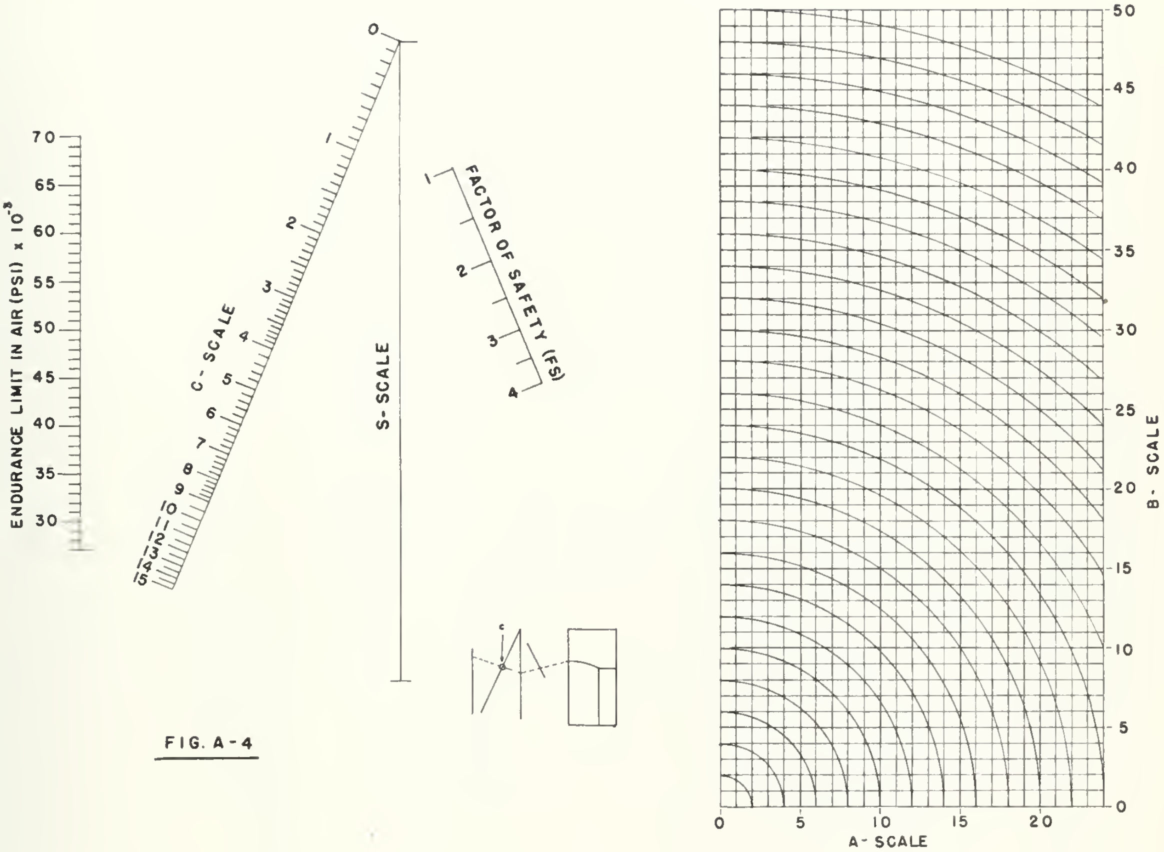

ENDURANCE LIMIT IN AIR (PSI) x 10⁻³
C - SCALE
S - SCALE
FACTOR OF SAFETY (FS)
A - SCALE
B - SCALE

FIG. A-4

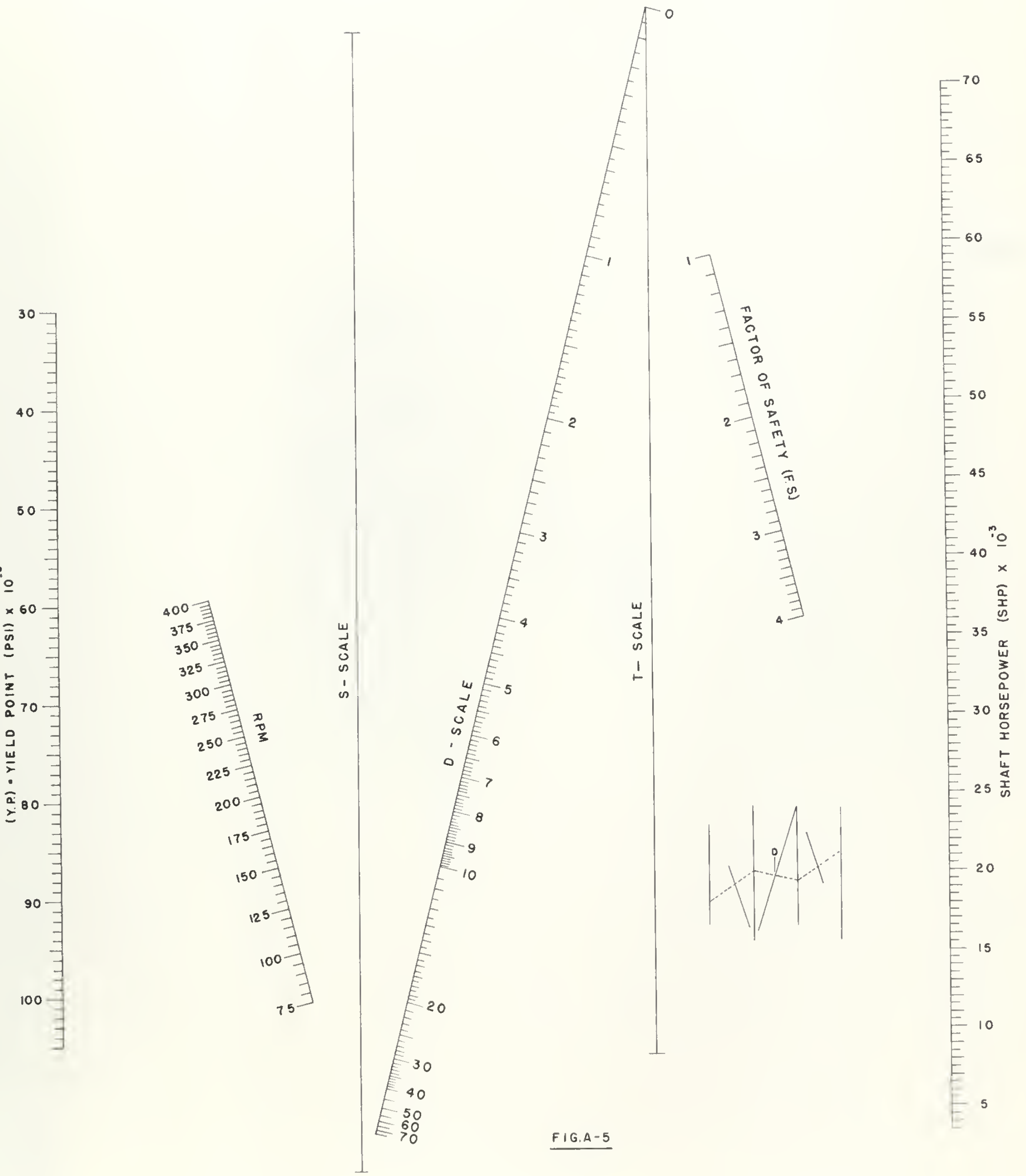

(Y.P.) • YIELD POINT (PSI) x 10⁻³
RPM
S - SCALE
D - SCALE
T - SCALE
FACTOR OF SAFETY (F.S.)
SHAFT HORSEPOWER (SHP) x 10⁻³

FIG. A-5

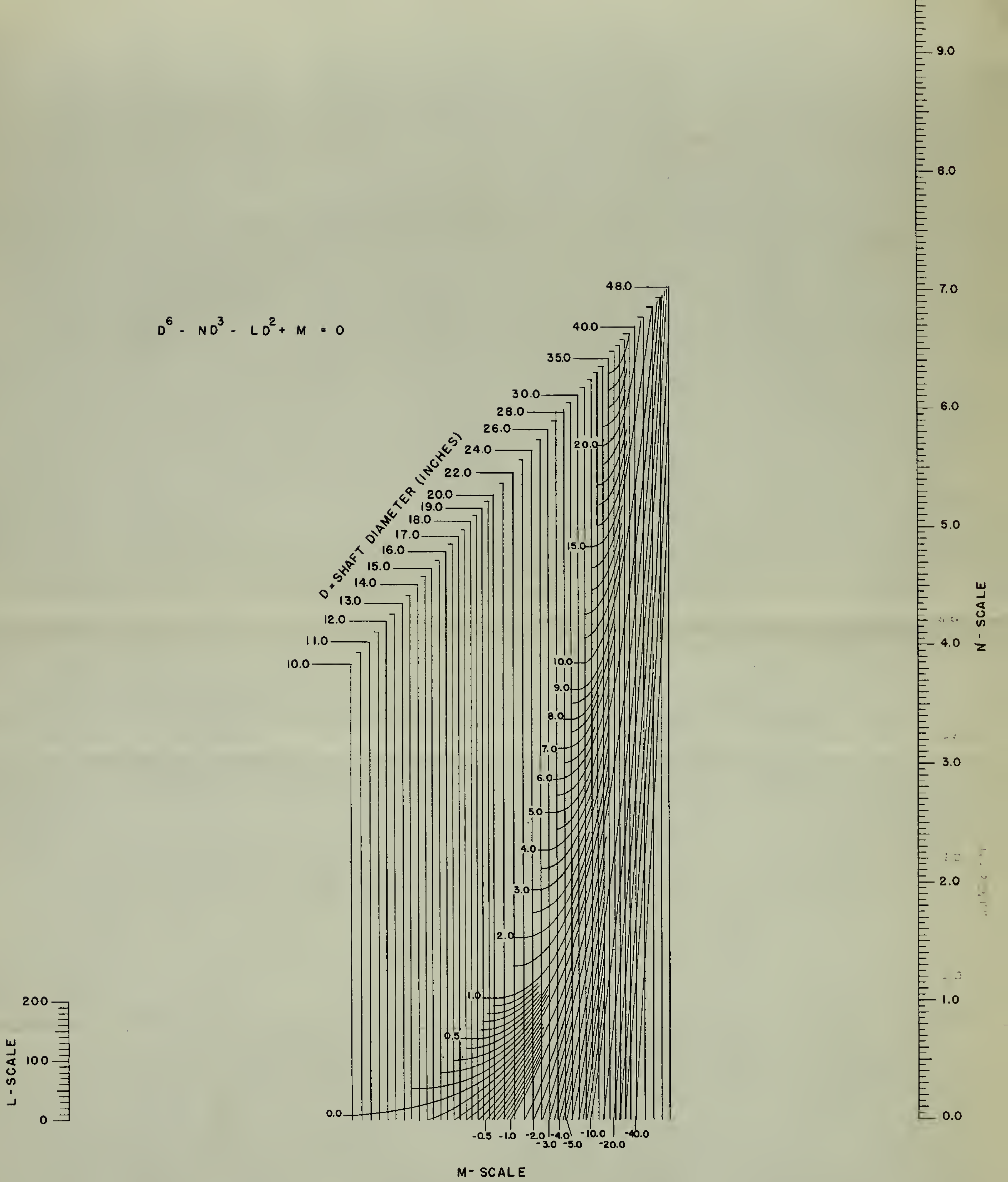


FIG. A-6

APPENDIX B

SHAFTING SYSTEMS COMPUTER PROGRAM

The program used is basically the same program reported in references (4), (6) and (14). The only significant differences are modifications to input and output subroutines for use with the IBM-709 and the inclusion of a matrix inversion subroutine from the SHARE library. Based upon the author's experience with both the 704 and 709 programs the machine time for the 709 is approximately one-fifth that of the 704.

The adaptation of the program to the 709 would not have been possible without the copy of the 704 program so generously furnished by Mr. H. C. Anderson of The General Electric Company.

Reference (14) or Appendix G of reference (6) gives a complete write-up of the program as used with the IBM-704 machine. This write-up is applicable to the IBM-709 program with the exception of the operating instructions. In the case of the operating instructions the 709 program is designed for operation using the MIT Computation Center's FMS system. Unfortunately in both references on the seventh page

of the section headed "THEORY" there appears an error which could confuse the user. This error is in the matrix which appears on that page and in the subsequent use of the matrix inversion. For the convenience of any potential user the corrected form starting at the middle of the page should be:

"All these equations can therefore be put into matrix form:

$$
\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ . \\ \delta_n \\ 0 \\ 0 \end{bmatrix}
=
\begin{bmatrix}
1 & x_1 & 0 & 0 & 0 & . & . & . & . \\
1 & x_2 & d_{12} & 0 & 0 & 0 & . & . & . \\
1 & x_3 & d_{13} & d_{23} & 0 & 0 & . & . & . \\
1 & x_4 & d_{14} & d_{24} & d_{34} & 0 & . & . & . \\
. & . & . & . & . & . & . & . & . \\
1 & x_n & d_{1n} & d_{2n} & d_{3n} & & & & \\
0 & 0 & 1 & 1 & 1 & 1 & . & . & 1 \\
0 & 0 & x_1 & x_2 & x_3 & & & & x_n
\end{bmatrix}
\times
\begin{bmatrix} \Delta_o \\ \theta_o \\ R_1 \\ R_2 \\ . \\ . \\ . \\ R_n \end{bmatrix}
$$

If we now take the inverse of the coefficient matrix and multiply both sides of the above matrix equation gives:

Letting a_{ij} represent the elements of the inverse matrix

$$\begin{bmatrix} \Delta_o \\ \theta_o \\ R_1 \\ R_2 \\ \cdot \\ \cdot \\ \cdot \\ R_n \end{bmatrix} = \begin{bmatrix} a_{11} a_{12} a_{13} a_{14} a_{15} & \cdot & \cdot & \cdot & a_{1m} \\ a_{21} a_{22} a_{23} a_{24} a_{25} & \cdot & \cdot & \cdot & a_{2m} \\ a_{31} a_{32} a_{33} a_{34} a_{35} & \cdot & \cdot & \cdot & a_{3m} \\ a_{41} a_{42} a_{43} a_{44} a_{45} & \cdot & \cdot & \cdot & a_{4m} \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{m1} a_{m2} a_{m3} a_{m4} a_{m5} & \cdot & \cdot & \cdot & a_{mm} \end{bmatrix} \times \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \cdot \\ \delta_n \\ 0 \\ 0 \end{bmatrix}$$

The first two rows of the inverted matrix give the deflection and slope conditions at station 1 for a unit deflection at the associated intermediate point; i.e., a_{13} is the deflection at station 1 for a unit deflection at support number 3. The remainder of the matrix gives the reactions for a given deflection; or the INFLUENCE numbers. For example, the element a_{34} is the change in reaction at support number 1 for a unit deflection at support number 4. (The unit deflection as scaled in the program is 0.001")

This means that assuming no external forces or weight:

$$R_1 = \delta_1 a_{31} + \delta_2 a_{32} + \delta_3 a_{33} \; . \; . \; . \; . \; + \delta_n a_{3n}$$

$$\theta_o = \delta_1 a_{21} + \delta_2 a_{22} + \delta_3 a_{23} \; . \; . \; . \; . \; + \delta_n a_{2n}$$

The remainder of the program write-up of the two references is correct.

Statistical data, as obtained from the computer calculation, for the cases studied is on file with the Department of Naval Architecture.

APPENDIX C

SELECTION OF SHAFT SYSTEMS FOR STUDY

1. It was necessary to synthesize groups of typical shaft system components from statistical data. If lineshaft and tailshaft diameter, shaft horsepower, and RPM are known it is possible to use the information in reference (6) for this purpose. The four reference parameters were selected in the following manner.

Lineshaft and Tailshaft Diameters - A series of shaft diameters, ranging from 10 to 30 inches inclusive, was arbitrarily specified. In order to reduce the number of cases to be studied and to simplify the study process, it was assumed that both the tailshaft and lineshaft would be of the same diameter. This assumption leads to an inefficient design in any real design problem since the lineshaft will generally be subjected to smaller stress levels than the tailshaft. It is possible to use smaller lineshaft diameters than tailshaft diameters because of this. However by choosing a single overall diameter which is strong enough to carry the applied loads on the tailshaft, a more conservative overall design is specified. Furthermore, since any alignment criteria is dependent on shaft stiffness and

bearing loading, a larger diameter will tend to make predicted values of minimum span lengths somewhat larger than those actually required. One of the objectives of the thesis study was the establishment of a minimum allowable span criteria. In this respect the use of larger diameters will result in a slightly conservative criteria. Solid shafts only were studied in the interest of simplicity.

RPM Selection - Values of RPM were arbitrarily chosen within the range of values found in present day ships.

Shaft Horsepower - For each combination of RPM and diameter a corresponding value for SHP was calculated. The calculation was made through an application of the following empirical formula from reference (13).

$$D = 0.95 \sqrt[3]{\frac{64 \text{ SHP}}{\text{RPM}}}$$

or

$$\text{SHP} = \left(\frac{D}{0.95}\right)^3 \frac{\text{RPM}}{64}$$

In this manner some measure of correlation was achieved between the entering parameters for use with the statistical data. Thus it was possible to obtain dimensions for a propeller and reduction gear which were compatible with each other and the assumed shaft diameter.

2. Once shaft diameter, RPM and SHP were known the following specific information was obtained for each shaft system.

Reduction Gear Dimensions

a. Gear shaft diameter - D_{gs}

b. Gear weight - W_g

c. Equivalent gear diameter - D_h

This is the diameter of a solid steel shaft which has a stiffness, in bending, equal to the stiffness of the reduction gear unit.

d. Length between gear support bearings - L_b

e. Length of bull gear face - L_f

f. Concentrated gear weight - W_g

When an equivalent gear diameter is used to replace the stiffness of the reduction gear, the equivalent shaft will have a smaller weight than the total weight of the gear unit. This difference in weight can be calculated and added to the equivalent shaft as a concentrated weight.

$$\Delta W_g = W_g - \left(\frac{\rho\pi}{4}\right)D_h^2\, L_f$$

ρ = Density of Steel, lbs./in.^{3}

Propeller Dimensions

a. Propeller weight - W_p

b. Propeller overhang - L_p

This is the distance from the center of gravity of the propeller to the shaft support point in the after stern tube bearing. The support point was assumed one shaft diameter forward of the outer extremity of the stern bearing.

c. Mass moment of inertia factor for the propeller about its axis - $W_p r^2$

As noted in reference (12), the mass moment of inertia of a propeller can be approximated by the formula

$$I_x = \frac{W_p r^2}{2g} \quad ; \quad r = \text{radius of gyration of the propeller}$$

3. With the above data a series of representative shaft systems was available for study. The study system parameters are shown in Table III. It should be noted that each of the final synthesized systems was checked in accordance with the strength criteria of Appendix D. This was done to ensure the adequacy of the arbitrary shaft diameters in light of the corresponding propeller and reduction gear dimensions. The only unknown information still to be determined for each system is allowable span lengths.

NOMENCLATURE PECULIAR TO APPENDIX D

A = Shaft cross-section area = $\frac{\pi D^2}{4}$, in^2.

E = Modulus of Elasticity = 29×10^6, psi.

f = Frequency of Lateral Vibration, cpm.

g = Acceleration due to gravity = 386 $\frac{16\ f\text{-}in}{16m - sec^2}$

I = Moment of inertia of shaft in bending = $\frac{\pi D^4}{64}$, in.^{4}

I_x = Mass moment of inertia of propeller about a diameter (increased 60% for entrained water) = $\frac{1.6 W_p r^2}{2g}$

J = Polar moment of inertia of shaft = $\frac{\pi D^4}{32}$, in.^{4}

L = Length between tailshaft support points, inches.

L_e = Length of lineshaft spans, inches.

M_t = Torsional Moment = $\frac{33{,}000 \times 12 \times SHP}{2\pi\ RPM}$, in.-lbs.

M_{max} = Bending Moment, in-lbs.

m = Propeller mass (increased 30% for entrained water) = $1.3 \frac{W_p}{g}$

T = Thrust = $\frac{33{,}000 \times SHP \times p.c.}{101.34\ V_{KTS}\ (1-t)}$

p.c. = Propulsive coefficient.

t = Thrust deduction.

V_{KTS} = Speed in knots.

SHP = Shaft horsepower.

w = Weight of shaft per inch = $\rho\frac{\pi D^2}{4}$

u = Shaft mass per inch = $\frac{\rho}{g}\,\frac{\pi D^2}{4}$

ρ = Density of steel = 0.282 lbs./in.^{3}.

APPENDIX D

STRENGTH AND VIBRATION REQUIREMENTS

1. The usual design process is concerned with the provision of adequate shaft diameters for required strength, and limits on the maximum length between supports to preclude the existence of vibration criticals in the range of operating RPM. An application of a strength and vibration criteria such as that outlined in reference (10) will satisfy these requirements. In the development of a minimum span criteria consideration of strength and vibration requirements do not enter directly. However a check had to be made on the compatibility of maximum and minimum span criteria; i.e., the minimum span must not be greater than the maximum allowable span required by strength and vibration considerations.

It was also necessary to make a direct shaft strength calculation for each of the synthesized study systems. This was done to ensure a large enough shaft cross-section to carry the loads of the various components.

2. The maximum bending stress occurs at the support point in the after stern tube bearing. It is caused by

the large overhung propeller weight and the effect of thrust eccentricities. All of the other basic stresses are common to the entire shaft length. Thus a shaft cross-section of sufficient size to carry the stresses at the after support point should be adequate for the remainder of the system. For each of the synthesized shaft systems, equation (20) was applied at that support point to check the adequacy of the shaft diameter. To apply equation (20) it was necessary to compute values for the various steady and alternating components.

<u>Steady Shear Stress</u>

$$S_s = \frac{M_T D}{2J} \quad ;$$

or

(1d) $$S_s = \frac{3.22 \times 10^5}{D^3}\left(\frac{SHP}{RPM}\right)$$

<u>Steady Compressive Stress</u>

$$S_c = \frac{T}{A}$$

The following parameters were assumed for all cases:

Propulsive coefficient, p.c. = 0.65

Speed in knots, $V_{kts.}$ = 20

Thrust deduction, t = 0.2

or

$$\text{(2d)} \quad S_c = 16.85 \left(\frac{RPM}{D^2}\right)\left(\frac{SHP}{RPM}\right)$$

Alternating Shear Stress

$$\text{(3d)} \quad S_{sa} = 0.05\ S_s = \frac{1.61 \times 10^4}{D^3}\left(\frac{SHP}{RPM}\right)$$

In all cases it was assumed that the alternating component of shear stress would be equivalent to 5% of the steady component.

Alternating Bending Stress

$$S_b = \frac{M_{max}\ D}{2I} \quad ;$$

For the after support point it was assumed that M_{max} was made up of the following parts:

$W_p L_p$ = Moment caused by propeller overhang

$\rho \frac{\pi D^2 L_p^2}{8}$ = Moment caused by shaft overhang

$M_{oc} = 2W_p L_p$, additional moment from thrust eccenticity

or

$$S_b = \frac{10.2}{D^3}\left(3W_p L_p + \rho \frac{\pi D^2 L_p{}^2}{8}\right)$$

For all cases the following data, with reference to stress concentration factors and type of shaft material,

was specified:

Class Bs steel:

Yield Point, Y.P. = 40,000 psi

Fatigue Limit, F.L. = 34,000 psi

Stress concentration factors:

Bending, k_b (at keyway) = 1.0

Torsion, k_t (at keyway) = 1.9

Upon inserting, in equation (20), the values computed from the above equations and assumptions, a safety factor for each of the basic study systems was computed. A shaft diameter giving a safety factor of approximately 2 was considered satisfactory. The results of these calculations are listed in Table V.

3. Allowable maximum tailshaft lengths were estimated through application of the following equation (11) for calculating frequency of lateral vibrations.

$$f = \frac{30}{\pi}\sqrt{\frac{EI}{I_x(L_p + \frac{L}{3}) + mL_p^2(\frac{L_p}{2} + \frac{L}{3}) + \mu(\frac{L_p^4}{8} + \frac{L\,L_p^3}{9} + \frac{7L^4}{360}}}$$

or upon rearranging

$$\text{(5d)} \quad \frac{7}{360}\mu L^4 + \frac{L}{3}\left[(I_x + mL_p^2) + \frac{L_p^3\mu}{3}\right] = \frac{900EI}{\pi^2 f^2} - \frac{L_p}{2}\left[2I_x + mL_p^2\right] - \frac{\mu L_p^4}{8}$$

The above equation was solved for length, L, for each study system via a trial and error method. These results are tabulated in Table V.

4. In the lineshaft region it was possible to calculate a maximum allowable span based on strength requirements for the given shaft diameter. Equation (20) can again be applied. Values for the steady shear, steady compressive, and alternating stresses as calculated in the shaft sizing procedure can be used directly. However it is necessary to recalculate a value for the alternating bending stress. The bending stress in the lineshaft was calculated by assuming that each lineshaft span acts like a built-in beam carrying a uniformly distributed load. The accuracy of this assumption was verified in several instances. It was found that an actual shaft span has a bending moment, at the shaft supports, within $\pm 8\%$ of that predicted through application of the built-in beam formula. Since oversize lineshaft diameters are required by the original assumption of a single shaft diameter, this was not felt to be an excessive error. Thus for the lineshaft alternating bending stress may be approximated by:

$$S_b = \frac{M_{max}\, D}{2I} \quad ; \quad M_{max} = \frac{w\, L_e^2}{12}$$

L_1 = This is the lineshaft span length

or

$$(6d) \qquad S_b = \frac{L_e^2}{D} \times 0.188$$

Examination of equations 1d, 2d, 3d, and 6d indicates only the alternating bending stress component is a function of span length. Inserting those equations into equation (20) and rearranging results in the following.

$$(7d) \quad L_e^2 = \frac{D}{0.188k_b}\sqrt{\left\{F.L.\left[\frac{1}{F.S.} - \frac{1}{Y.P.}\left(\frac{SHP}{RPM}\right)\left(\frac{1}{D^2}\right)\left(\frac{41.5x10^{10}}{D^2}+284RPM^2\right)^{\frac{1}{2}}\right]\right\}^2 - \left\{\frac{3.22x10^4\ b_t SHP}{D^3\ RPM}\right\}^2}$$

For all cases the following data was assumed:

Class B Steel[10]

Yield Point = 30,000 psi

Fatigue Limit = 27,000 psi

Stress concentration factors;

Bending, k_b = 2.0

Torsion, k_t = 1.9

Required safety factor, F.S. = 1.75

Inserting these assumptions in equation (7d) yields:

$$(8d)\quad L_e^2 = \frac{D}{0.376}\sqrt{\left\{0.9\left[1.72\times10^4 - \frac{SHP}{RPM}\left(\frac{1}{D^2}\right)\left(\frac{415\times10^{10}}{D^2} + 284RPM^2\right)^{\frac{1}{2}}\right]\right\}^2 - \frac{3.74\times10^9}{D^6}\left(\frac{SHP}{RPM}\right)^2}$$

This equation was solved for each of the study systems and the estimated values of maximum lineshaft span based on strength are listed in Table V.

5. An attempt was made to estimate maximum lineshaft span lengths from vibration considerations. Usually an application of the following equation can be expected to give a close approximation to the fundamental whirling critical frequency.[5]

$$f = 187.7\sqrt{\frac{M.y}{M.y^2}}$$

y = deflection, inches

M = weight of shaft corresponding to deflection y, lbs./mass

f = critical frequency, cpm

Unfortunately this equation is not suitable for simple algebraic manipulation to express span length. For a multiple supported shaft, carrying distributed loads, a separate de-

flection curve must be calculated. Span length enters only through calculation of the deflection curve.

A simpler, though more gross approximation, can be made. Each lineshaft span was considered as a simply supported beam carrying a distributed lateral load and subjected to a compressive end thrust. For this beam configuration it is possible to derive an equation expressing critical frequency as a function of span length from the differential equation for lateral vibration.(6)

$$f = \frac{30}{\pi}\sqrt{\frac{EI\,\pi^4\,g}{w\,L_e^4} - \frac{T\,\pi^2\,g}{w\,L_e^2}}$$

f = cpm.

This equation was then manipulated to achieve a simpler form for length estimating. To account for unknown elasticity of the bearing supports critical frequency was specified equal to 2.5 times RPM.

Thus

$$\text{(9d)}\quad L_e^2 = \frac{3.81\,D}{RPM}\sqrt{2.42 \times 10^{11} - \frac{6.4\times10^4\,SHP \times L_e^2}{D^4}}$$

A series of simple trial and error computations resulted in predicted estimates of maximum allowable span length. These results are listed in Table V.

6. It should be reemphasized that the values listed in Table V are not generally applicable to all possible shaft designs. They are only estimates for the synthesized systems studied in this thesis.

With respect to the critical problem of tailshaft sizing, the nomograms of Appendix A can be used with any combination of system parameters to estimate a satisfactory tailshaft diameter. However no attempt was made to provide a maximum span length criteria of general applicability. The values calculated are only guides used to indicate limits for the minimum span length criteria. Relatively small changes in propeller dimensions and/or propeller overhang from those used in the thesis study could have a significant effect on increasing or decreasing maximum tailshaft length. Changes in material specification, required safety factor, diameter, or a combination of changes can result in a different maximum lineshaft span length. A study of the effect of changes in the various design parameters would be of definite value. Such a study was outside the aims of the present thesis investigation. However, the equations derived for use in setting maximum limits on span length in the preceding paragraphs may be of some assistance in future shaft design problems.

TABLE V

TAILSHAFT SAFETY FACTOR AND ALLOWABLE SPAN LENGTH ESTIMATES

Shaft Diameter, inches	10	12	14	16	18	20	22	24	26	28	30
Tailshaft: Factor of Safety	1.96	2.04	2.09	2.1	2.06	1.99	2.15	2.07	2.01	2.01	1.95
Max. Span Length, inches	600	653	675	690	728	770	665	682	712	717	725
L_{max}/D	60	54.3	48.2	43.1	40.4	38.5	30.8	28.4	27.3	25.6	24.1
Lineshaft: Max. span length, inches (from strength)	351	385	420	449	476	498	526	552	574	589	609
L_{emax}/D	25.1	32.1	30.0	28.0	26.5	24.9	23.9	23.0	22.1	21.0	20.3
Max. Span Length, inches (from vibrations)	469	513	556	542	574	606	525	547	768	573	592
L_{emax}/D	46.9	42.7	39.8	33.9	31.8	30.3	23.9	22.8	21.8	20.4	19.7